MODIFIED TTO LAW FOR MATERIAL MIXTURES WITH APPLICATION TO 6-PARAMETER NONLINEAR SHELL ANALYSIS

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1. Introduction

Plates and shells build from Functionally Graded Materials (FGMs) are commonly analysed in elastic and elasto-plastic range. Material mixture law must define every material parameter as a function of relative volumes of constituents. In Tamura, Tomota and Ozawa (TTO) mixture law [1] it is assumed that mixed or metallic material behaviour is elasto-plastic with linear hardening, while for pure ceramics material is elastic. This material law was enhanced to nonlinear hardening curve in paper [2], where also validation with experimental tests was conducted. Validation resulted in determining parameters for Ti/TiB mixture.

In the present research, modification of TTO law is proposed. The motivation follows from the fact that the original TTO model assumes that even the smallest inclusion of elasto-plastic metal constituent in the elastic ceramic matrix, rapidly changes material behaviour, from linear to inelastic. The proposal presented herein ensures smoothing such a rapid change and thereby introduce hyperbolic-like yield point function. The proposed form of equation is implemented within the framework of nonlinear 6-parameter shell theory with drilling rotation [3]. Specifically, an own FEM Fortran code named CAM [3] for nonlinear shell analysis is used. In formulation of the constitutive matrix for shell finite element we assume Cosserat-like plane stress at each integration point in the shell reference surface and return mapping algorithm as described in [4] for details.

2. Material law

Let (*c*) stands for ceramic and let (*m*) for metal constituent. Let the shell section is ceramic rich on the top surface $(+h^+)$ and metal rich on the bottom surface $(-h^+)$. The power law

(1) $$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n, \quad V_m = 1 - V_c, \quad n \geq 1$$

describes the changes of material constituents in the thickness direction *z*. Here *n* denotes the power-law exponent. The effective Young modulus *E*(*z*), the effective Poisson ration *v*(*z*), the ratio of stress to strain transfer *q* and the linear hardening modulus *H*(*z*) are described by original TTO law. Since in the current theory Cosserat plane stress is assumed in each shell layer, its additional effective constants are defined as

(2) $$l(z) = l_c V_c + l_m V_m, \quad \kappa(z) = G(z)\frac{N^2}{1-N^2},$$

where *l*(*z*) is characteristic length and $0 < N < 1$ is the coupling number. In this paper we propose to modify the original TTO equation for the yield stress, namely

(3) $$\sigma_Y(z) = (1-k_Y)\sigma_{Ym}\left(V_m + \frac{q+E_m}{q+E_c}\frac{E_c}{E_m}V_c\right) + k_Y\frac{\sigma_{Ym}}{1-V_c}.$$

That is, we postulate the existence of the scalar k_Y such that $1 \geq k_Y > 0$. Equation (3) holds only if $V_c < 1$. When $V_c = 1$ the material is purely ceramic and does not possess the yield limit. Fig.1 displays the idea how variations of *q* and k_Y from equation (3) influence the stress vs. strain curve in original and modified TTO material law.

3. Numerical example

As a numerical example, equilibrium paths for axially compressed plate are shown in Fig. 2. Geometry and material data [5, 6] are: a = 200, b = 100, b/t = 40, n = 5.0, E_c = 375000, ν_c = 0.14, l_c = 0.005, E_m = 107000, ν_m = 0.34, l_m = 0.005, H_m = 4600, σ_{Ym} = 450, q = 4500, $\kappa = G$. Reference solution, obtained with original TTO law is taken from [6]. Both parametric studies reveals growth of limit load along with the rise of values of parameters. There is no major qualitative change observed in both cases.

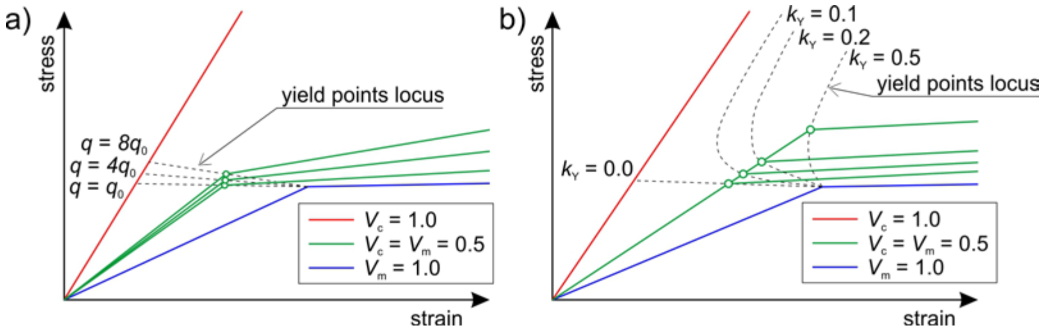

Fig. 1. Parametric analysis a) q in original TTO law b) k_Y in modified TTO law.

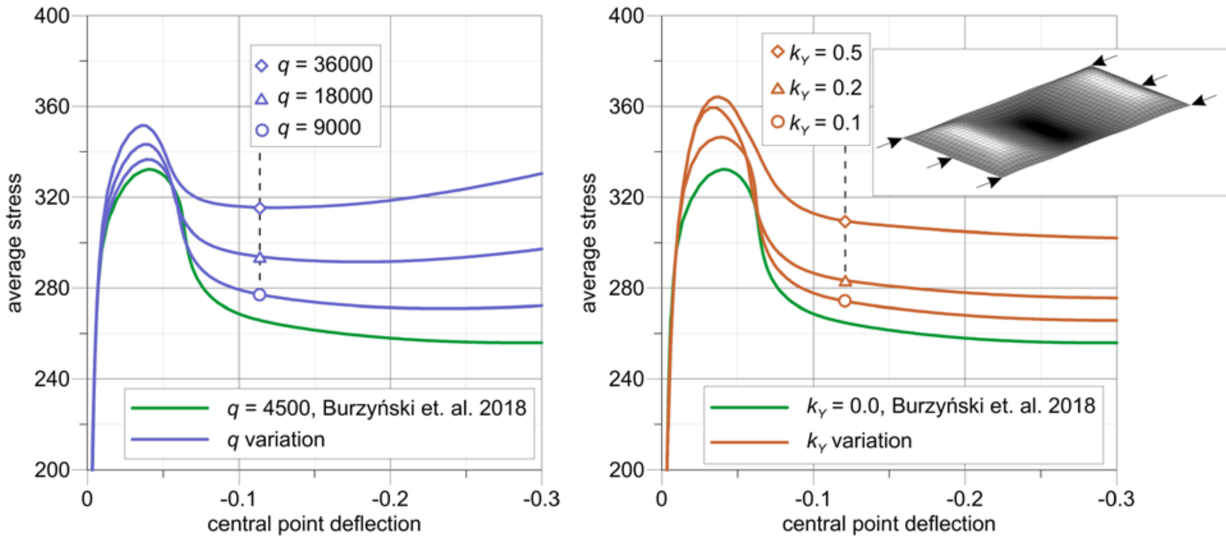

Fig. 2. Parametric analysis in compressed plate a) q variation b) k_Y variation.

Acknowledgments The authors acknowledge financial support from the grant No UMO 2015/17/B/ST8/02190, financed by National Science Centre Poland.

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