

# A NUMERICAL MODEL OF SINTERING PROCESSES AT MACROSCOPIC LEVEL

K. Wawrzyk, P. Kowalczyk, J. Rojek, and S. Nosewicz

*Institute of Fundamental Technological Research, Polish Academy of Sciences  
Pawińskiego 5B, 02-106 Warsaw, Poland*

*e-mail: kwawrzyk@ippt.pan.pl*

## 1. Introduction

This paper presents modelling of double-phase powder sintering processes at the macroscopic level. In particular, its constitutive formulation, numerical implementation and numerical simulations are described. Numerical tests were carried out for a cylindrical specimen under uniaxial pressure and are compared against the microscopic model results. The model has been developed within the framework of a MUSINT project which is carried on in Institute of Fundamental Technological Research, Warsaw, Poland. The overall objective of the MUSINT (Multiscale numerical modelling of sintering processes) is development of numerical models allowing us to analyse at various scales manufacturing processes employing sintering as the main technological stage.

## 2. Constitutive model of single- and double-phase sintering

Sintered material is treated as continuous medium at the macro scale. Its constitutive model is tightly related to observations done at the micro scale (single grain level) and fully consistent with the model describing the microscopic mechanical phenomena of the sintering process. Such a model, proposed by Nosewicz [5], is schematically depicted in Fig.1 and is introduced as an author's original extension of viscous model. The rheological model of sintering presented in Fig.1 is enriched by adding elastic and thermal component to the standard viscous model [3] [2] [1]. By analysis of mass transport and stresses at the grain boundary between two sintered particles the following equation for the particle interaction during sintering is obtained [4]:

$$(1) \quad F_n = \frac{\pi a^4}{8D_{eff}} v_{rn} + \pi \gamma_s \left[ 4r \left( 1 - \cos \frac{\Psi}{2} \right) + a \sin \frac{\Psi}{2} \right]$$

where  $F_n$  is the normal force between two particles,  $v_{rn}$  - the normal relative velocity,  $r$  - the particle radius,  $a$  - the radius of the interparticle boundary,  $\Psi$ - the dihedral angle,  $\gamma_s$  - the surface energy.

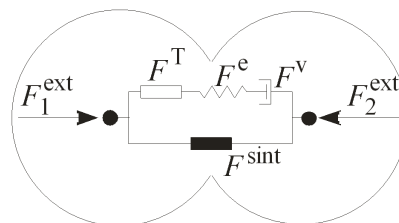


Figure 1: Rheological scheme of thermo-viscoelastic model [5]

At the macro scale sintered material is treated as continuous medium. Its constitutive model is tightly related to observations made at the micro scale (single grain level) and fully consistent with microscopic mechanical model described in the above section. The macroscopic model is derived with the use of methodology known from works of e.g. Zhang [6]. The total strain rate is a sum of three parts: thermal, elastic and viscous, as follows:

$$(2) \quad \dot{\epsilon} = \dot{\epsilon}^{th} + \dot{\epsilon}^e + \dot{\epsilon}^{visc}$$

Thus, the constitutive equation has the form:

$$(3) \quad \dot{\boldsymbol{\sigma}} = \mathbf{D} \left( \dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{th} - \dot{\boldsymbol{\varepsilon}}^{visc} \right) + \dot{\boldsymbol{\sigma}}^s$$

where thermal strain, viscous strain rate, sintering stress is given as follows:

$$(4) \quad \boldsymbol{\varepsilon}^{th} = \alpha T \mathbf{I}$$

$$(5) \quad \dot{\boldsymbol{\varepsilon}}^{visc} = \frac{\boldsymbol{\sigma}'}{2\eta_s} + \frac{tr(\boldsymbol{\sigma}) - 3\sigma_s}{9\eta_b} \mathbf{I}$$

$$(6) \quad \boldsymbol{\sigma}^s = \sigma_s \mathbf{I}$$

$$(7) \quad \sigma_s = \frac{\beta \gamma_s \rho^2}{r_0}, \rho = \hat{\rho} e^{-(\varepsilon_x + \varepsilon_y + \varepsilon_z)}$$

All parameters in the above equations — viscosity  $\eta_s$ ,  $\eta_b$ , initial relative density  $\hat{\rho}$ , surface energy  $\gamma_s$  etc. — are related to microscopic material properties. Some modifications of the material model parameters are required to simulate the sintering process of two-phase powder. The model of sintering of the two-phase powder NiAl-Al<sub>2</sub>O<sub>3</sub> is based on the mixture theory which is used to predict behaviour of multiphase systems and the basic assumption is that, at any instant of time, all phases are present at every material point. In general, for some material properties it provides theoretical upper and lowerbounds on properties - Voigt and Reuss model, respectively. Formulas for parameter  $W$  by Voigt and Reuss model is presented in Eqs. (8) and (9) respectively:

$$(8) \quad W = V_1 W_1 + V_2 W_2$$

$$(9) \quad W = \left( \frac{V_1}{W_1} + \frac{V_2}{W_2} \right)^{-1}$$

where:  $V_i$  and  $W_i$  are the volume fraction and material property of  $i$ -th constituent, respectively. In this work, theoretical upper and lowerbound were used to predict surface energy  $\gamma_s$  and shear or bulk viscosity  $\eta_s$  and  $\eta_b$ . The last two parameters are subject to investigations in our research and will be presented with the numerical results at the conference.

### 3. References

**Acknowledgments** This work has been financed from the funds of Polish National Science Centre (NCN) awarded by decision no. DEC-2013/11/B/ST8/03287.

#### References

- [1] R.L. Coble. Sintering of crystalline solids. intermediate and final state diffusion models. *Journal of Applied Physics*, 32:787–792, 1961.
- [2] D.L. Johnson. New method of obtaining volume, grain boundary, and surface diffusion coefficients from sintering data. *Acta Metallurgica*, 36:223–229, 1998.
- [3] L.C. De Jonghe and M.N. Rahaman. Sintering stress of homogeneous and heterogeneous powder compacts. *Acta Metallurgica*, 36:223–229, 1998.
- [4] S. Nosewicz, J. Rojek, K. Pietrzak, and M. Chmielewski. Viscoelastic discrete element model of powder sintering. *Powder Technology*, 246:157–168, 2013.
- [5] S. Nosewicz, J. Rojek, K. Pietrzak, and M. Chmielewski. Discrete element modeling and experimental investigation of hot pressing of intermetallic nial powder. *Advanced Powder Technology*, 28:1745–1759, 2017.
- [6] R. Zhang. *Numerical simulation of solid state sintering of metal powder compact dominated by grain boundary diffusion*. The Pennsylvania State Univeristy, 1998.