AN ELASTIC-PLASTIC TORSION OF OPEN PROFILES MADE WITH FUNCTIONALLY GRADED MATERIAL - NUMERICAL EXPERIMENT BASED ON MESHLESS METHODS

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1. Introduction

Elastic-plastic torsion of a bar is an important engineering problem, still discussed in literature. Special class of the bars is under considerations, i.e. we will consider prismatic bars and profiles with open cross-section. Moreover, it is assumed that the bars are made with functionally graded materials.

Twisting of prismatic bars made with homogeneous and isotropic materials have been undertaken by many authors [5; 10; 9; 11]. In the last time, the case of inhomogeneous and/or anisotropic material is more often discussed in literature [2; 3; 12]. It is related to the research of functionally graded materials (FGMs), designed for special engineering applications including aircraft,

aerospace, automobile industry and medicine. Functionally graded materials are characterized by the continuous changes of their properties at least in one direction and this feature distinguishes them from the conventional composite materials [8]. In fact the concept of FGMs is inspired with materials occurring in nature, such as: bones, skin and bamboo [4]. These materials have functionally graded and hierarchical structure and they also have different architecture that results in orthortropic behaviour [1].

In this work the elastic-plastic torsion problem of bars made with FGMs is investigated. In the recent years a number of numerical methods of solving this problem have been developed and improved. It is a more complex issue than problem of elastic bar torsion and proposed numerical methods use different approaches.

2. Model of phenomenum

This is a boundary value problem, described by partial differential equation of second order with variable coefficients and appropriate boundary conditions. The problem is formulated for the Prandtl's stress function. Generally, when the shear flexibility modulus is an arbitrary function of cross-sectional coordinates, the analytical solution is not available.

We consider a functionally graded bar of an arbitrary and uniform cross-section. The axis Oz is parallel to the longitudinal axis of the bar and the bar is twisted by two couples of forces acting on its ends. It is assumed that there are no body forces and the bar is free from external forces on its lateral surface. There are no normal stresses on the frontal cross-section.

In case of functionally graded material characteristic G in the torsion equation is a shear modulus in all direction axis on the plane whose is normal in direction Z. The G is a function of geometrical variables, i.e. G(x,y). It is assumed that the G(x,y) is the continuous and differentiable function depending on geometrical coordinates x and y.

The problem is formulated in terms of the Prandtl's stress function and it is described by the equation [7], which is Poisson-like equation. The equation consists of Laplace operator of Prandtl's stress function. In the elastic state the inhomogeneous part of equation is equal to -2. For plastic state the governing equation consists of Laplace operator and another additional non-linear differential operator of the Prandtl's function.

For the formulation the boundary condition has to described. The boundary condition says that the Prandtl's function is equal to zero on the whole boundary.

The stresses are defined as proper partial derivatives of Prandtl's stress function.
The state of each point of considered cross-section of the bar is estimated by von Misses (Huber) hipothesis, using the stresses given by Prandtl's function.
Moreover, the hardening of the material is takes into account. The hardening phenomenum is described by nonlinear analytical function.

3. Meshless methods

The proposal of this paper is to solve problem of elastic-plastic torsion of bar made with FGM using numerical procedure based on one of the meshless methods. The used method is the Fundamental Solution Method (see [6]) supported by approximation by Radial Basis Function. Moreover, the iterative algorithm is applied to tread nonlinearities of the differential equation.
The results of numerical calculation are presented in the paper. The parameters of proposed numerical algorithm are chosen by experiment.
The postprocessing analysis is made, i. e. the stresses and effective plastic stress, effective strain, material flow are calculated and plotted.
The influence of material characteristics and parameters on the torsion, range of plastic deformations are investigated and discussed.

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