NONLINEAR FEA OF VIBRATION CONTROL OF PIEZOELECTRIC ROD-TYPE STRUCTURAL MEMBERS

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1. Introduction

This lecture addresses modelling and finite element analysis of the transient large amplitude vibration response of thin rod-type structures with piezoelectric layers. We present the analysis of two problems that are the FE analyses of a clamped semicircular ring and a smart circular arch subjected to a hydrostatic pressure. The presented results are based on the paper [1].

2. Geometrically nonlinear theory of plane piezolaminated rods and numerical method

We consider the planar deformation of a naturally curved plane rod with integrated piezoelectric layers. The analysis is based on the Bernoulli theory of plane deformations of beams, which is rich enough to accommodate the longitudinal extension and flexure neglecting the transverse shear strains. In the framework of the present theory, the deformation of the rod is completely determined by two components of the displacement vector $\boldsymbol{u}(s)$ at the rod axis. Within the through-the-thickness integration we introduce the normal stress resultant N and the bending couple M, which are work-conjugate to the 2D strain measures ε and κ. We formulate the virtual work principle

$$G[\boldsymbol{u},t;\delta\boldsymbol{u}] \equiv \int_{s_b}^{s_a} (\delta\varepsilon N + \delta\kappa M)\,\mathrm{d}s - G_e + G_b = 0 \tag{1}$$

for every kinematically admissible virtual displacement field $\delta\boldsymbol{u}(s)$ and every time $t > 0$. Here G_e and G_b are the work of inertia and external forces, respectively, $s \in [s_a, s_b]$ denotes the arc length coordinate along the undeformed rod axis. In what follows Eq. (1) serves as the basis for a finite element formulation.

For numerical calculations we use the 1D 2 – node C^1-element with four degrees of freedom at each node. Using the linearization and the standard FEM approximation procedures we get the classical incremental form of the equations of motion

$$\mathbf{M}\,\Delta\ddot{\mathbf{q}} + \mathbf{C}_t\,\Delta\dot{\mathbf{q}} + [\mathbf{K}_T - \mathbf{K}_L]\Delta\mathbf{q} = \Delta\mathbf{p} + \mathbf{j}_t\,, \tag{2}$$

where, $\mathbf{K}_T(\mathbf{q}_t)$ is the tangential stiffness matrix, $\mathbf{K}_L(\mathbf{q}_t)$ is the load matrix, $\mathbf{q}^{\mathrm{T}} = \{\mathbf{u}_{(1)}\,\mathbf{u}_{(2)}\ldots\mathbf{u}_{(a)}\ldots\}$, denote the global vectors of nodal displacements, and $\Delta\mathbf{p} \equiv \mathbf{p}_{t+\Delta t} - \mathbf{p}_t$ is the vector of load increments. Eq. (2) is solved in the time domain by the Newmark method of time integration, see [1] for details.

3. Numerical Results

We discuss the static and dynamic behaviour of two thin three-layered structures: a laminated semicircular ring shell and a circular arch with piezoelectric patches within the geometrically nonlinear range of deformation, see Figure 1. Some results of the control of the ring is given in Figure 2 whereas the buckling analysis of the arch is presented in Figure 3, see [1] for more details.

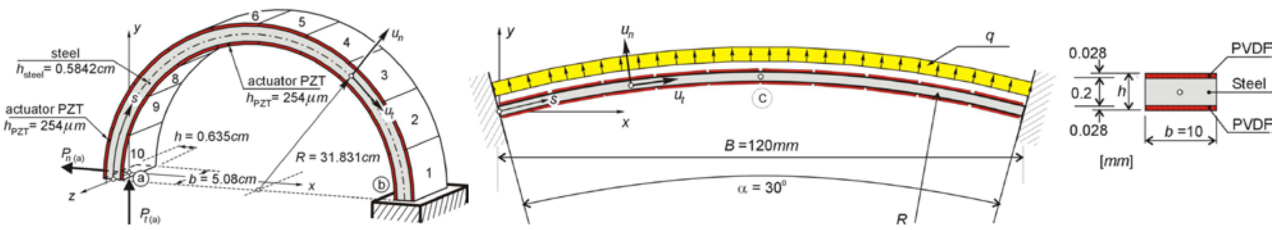

Figure 1: Laminated semicircular ring shell and circular arch with piezoelectric patches.

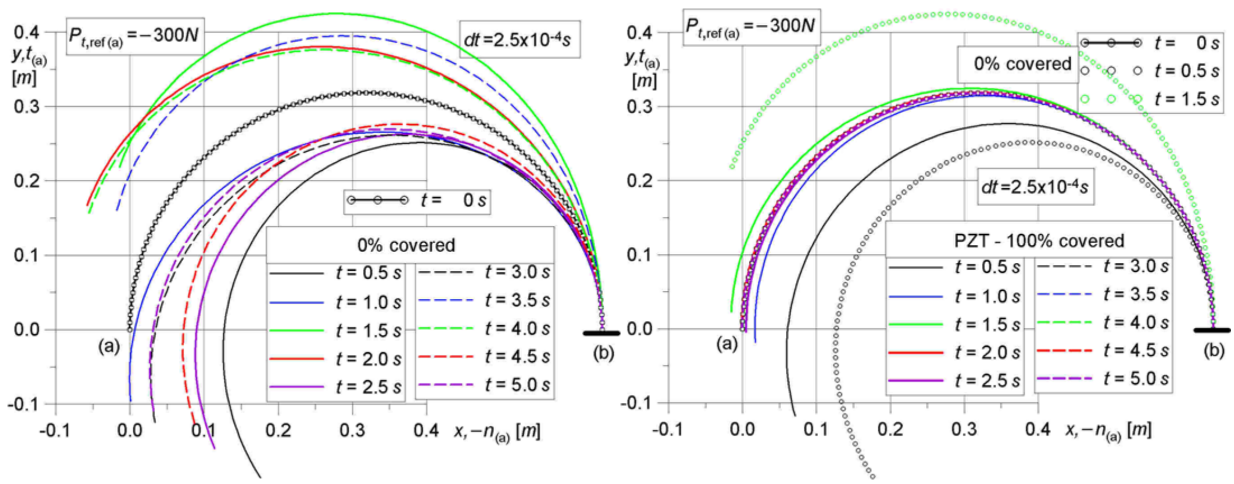

Figure 2: Uncontrolled (on the left) and controlled (on the right) responses of the ring tip, sequence of the deformed configurations.

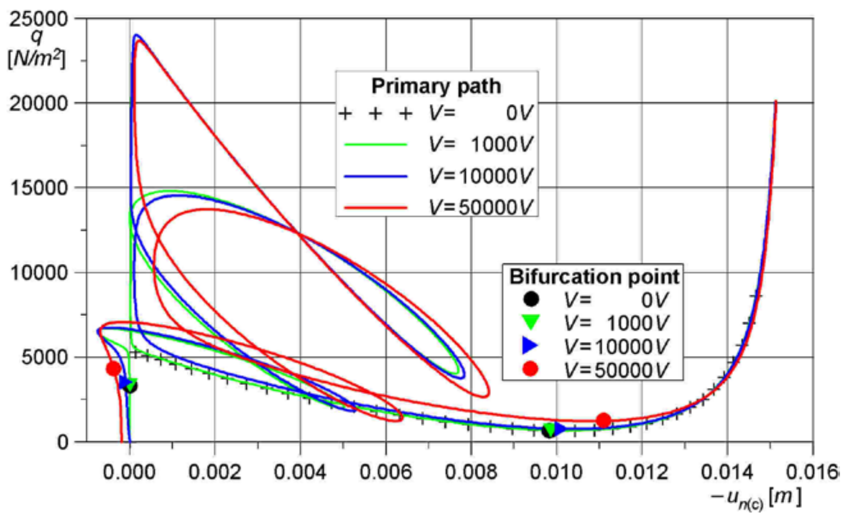

Figure 3: Load versus normal displacement in the middle point of the arch

References

[1] J. Chróścielewski, R. Schmidt, and V. A. Eremeyev. Nonlinear finite element modeling of vibration control of plane rod-type structural members with integrated piezoelectric patches. *Continuum Mechanics and Thermodynamics*. DOI: 10.1007/s00161-018-0672-4, 2018.