REGULARIZED LARGE STRAIN ELASTO-PLASTICITY: SIMULATION OF A PROPAGATIVE INSTABILITY

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1. Introduction

Softening of any kind (material, geometrical or thermal) can provoke the occurrence of a localized deformation mode in the form of a neck or a shear band. When a classical continuum description is employed this can involve an excessive mesh sensitivity of simulation results, which is caused by strains tending to localize in the smallest volume admitted by discretization. This pathological effect can be prevented by an upgrade of the constitutive description called regularization, involving either nonlocal averaging, spatial gradient dependence or time rate sensitivity.

The localization zones are often stationary and can lead to failure. On the other hand, one can encounter so-called propagative instabilities [1, 6], i.e. localized patterns which evolve in the loading process. One of such phenomena are Lueders bands, see for instance [3].

In this paper the thermal influences are neglected and attention is focused on phenomenological modelling. Two large strain elasto-plasticity models are used: the first one, presented for instance in [5], is extended towards viscoplasticity, the second one is based on [2] and gradient-dependent. Numerical analysis of a propagative instability of a Lueders type is performed for a two-dimensional configuration in plane strain conditions subjected to imposed tensile deformation [5].

2. Model description

The considered model is based on the multiplicative decomposition of the deformation gradient $\mathbf{F} = \mathbf{F}^e\mathbf{F}^p$. The state of the material is described by the Helmholtz free energy, decoupled additively in elastic and plastic hardening parts $\psi(\mathbf{b}^e, \gamma) = \psi^e(\mathbf{b}^e) + \psi^p(\gamma)$, where $\mathbf{b}^e$ is the elastic left Cauchy-Green tensor and γ is a scalar plastic strain measure.

The plastic process is governed by the yield condition $F_p(\boldsymbol{\tau}, \gamma) = f(\boldsymbol{\tau}) - \sqrt{2/3}\sigma_y(\gamma) \leq 0$. The equivalent stress function $f(\boldsymbol{\tau})$ is a Kirchhoff stress measure (e.g. J_2 Huber-von Mises type) and $\sigma_y(\gamma)$ denotes the yield strength which includes multi-linear strain hardening/softening. To regularize the model either a viscous term $\xi\dot{\gamma}$ is included in σ_y making the model rate-dependent (it is so-called consistency viscoplasticity [6] with viscosity parameter ξ) or isotropic degradation of plastic properties as in [2] is incorporated together with a gradient-enhancement.

3. Implementation and numerical tests

The numerical simulations are performed using symbolic-numerical packages *Ace* in *Wolfram Mathematica* environment [4]. Standard hexahedral elements with linear interpolation of all fields and *F-bar* modification are employed. Special attention is focused on the ability of the models to represent the propagative instability.

The adopted material model parameters are: Young's modulus E = 207 GPa, Poisson's ratio ν = 0.29, initial yield strength σ_{y0} = 450 MPa. For the yield strength softening is initially assumed with H_1=-0.01 E as the equivalent plastic strain grows from 0 to γ_1=0.15, then the yield strength is constant until γ_2=0.3, and hardening with H_2=0.005 E follows.

The dimensions of the plate are: L=0.1 m, S = 0.05 m, H=0.0025 m. To set the position of the incipient shear band a 10% reduction of the yield strength is assumed in one element at the lower left-hand corner of the configuration. The plate extension is specified by factor λ which scales the total imposed elongation of $0.4L$.

Figure 1 shows the diagrams of the sum of reactions versus the imposed elongation for the two regularized models. Figure 2 presents deformed meshes with the distributions of accumulated plastic strain γ. The left plots exhibit shear bands formed due to softening. The right ones show that they broaden and hardening induces a propagation of the plastic front while the plastic zone expands. The process is sensitive to the viscosity and weakly sensitive to the nonlocality present in the gradient model.

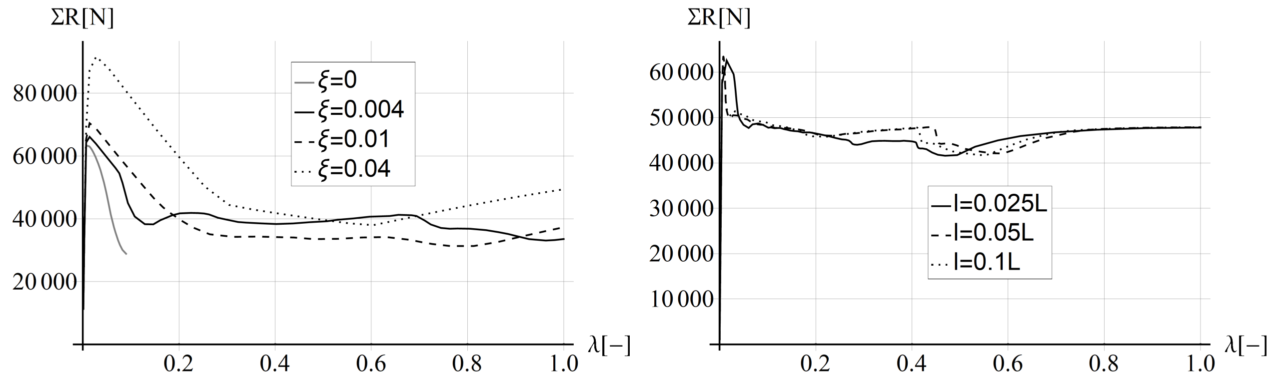

Figure 1: Force-extension diagrams for the viscoplastic model (left, different viscosities) and gradient-dependent model (right, different internal lengths)

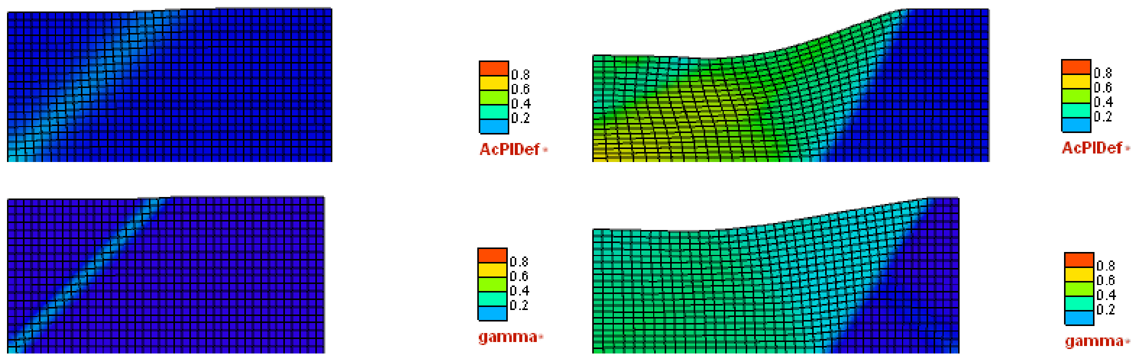

Figure 2: Distributions of equivalent plastic strain: first row - viscoplastic model with ξ=0.004 for two extension values λ = 0.08 (left) and λ = 0.66 (right); second row - gradient-enhanced model with internal length l = $0.1L$ for λ = 0.02 (left) and λ = 0.42 (right)

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