CONVERGENCE LIMIT OF A DEFORMABLE DISCRETE ELEMENT MODEL

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1. Problem statement

Despite successful use of discrete element method or DEM over a wide variety of engineering problems, the rigidity of particles assumed in classical formulation of DEM leads to incorrect behaviour in applications such as powder compaction at higher relative densities [1]. In our previous work [3], we have proposed an original formulation of DEM for deformable particles which we term as deformable discrete element method or DDEM. In-fact we have proven that the modelling capabilities of standard DEM can be enhanced by taking proper account of particle deformation. In order to incorporate deformability in discrete elements, the iterative scheme of our novel formulation results in an implicit relationship between contact forces and particle displacements. In presented work, the convergence limit for this implicit relationship is obtained analytically and verified numerically. The idea of DDEM is explained further.

2. Basic formulation of the deformable discrete element method

Referring to Fig.1 idea of DDEM can be explained as follows, under the uniform stress assumption a global deformation mode is introduced in particles which in-turn establishes new contact interaction due to reshaping of the particle and invokes force redistribution. Simultaneously, the modification in particle shape also leads to change in local interaction and hence alters the particle overlap from say, h to h_c (cf. Fig. 1). In this way, due to particle deformation, the contact in one point influences the contact interaction at other points and thus a non-local contact model evolves. This is the distinctive feature of DDEM formulation with respect to standard DEM where contacts are independent and do not influence each other. Considering the global deformation mode to deduce particle overlap and consequently the contact force gives an implicit relationship of the form,

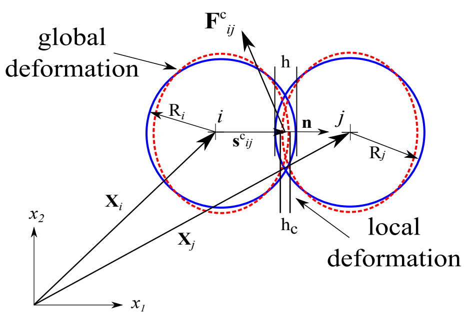

Figure 1: The idea of the deformable discrete element method (DDEM)

(1) $$\mathbf{F}_c^{(n)} = \mathbf{F}_c(\mathbf{u}^{(n)}, \boldsymbol{\epsilon}_p(\mathbf{F}_c^{(n)}))$$

where, the superscript n denotes the current time step and $\boldsymbol{\epsilon}_p$ indicate the strains instigated in particles due to contact force at current time step, $\mathbf{F}_c{}^{(n)}$. It will be shown that the implicit relationship of Eqn. (1) can be solved iteratively and corresponding relationship for the successive differences can be formulated as:

$$\mathbf{F}_c^{(n,k+1)} - \mathbf{F}_c^{(n,k)} = \mathbf{B}\,(\mathbf{F}_c^{(n,k)} - \mathbf{F}_c^{(n,k-1)})\,, \quad k >= 1\,. \tag{2}$$

where, $\mathbf{B}$ is certain matrix and $k-1$, k, $k+1$ represent the consecutive iterations for a given time step n. It must be noted though that the iterative solution may not always converge. In general, the convergence requires that for a certain matrix norm $\|\cdot\|$ we have, cf. [2]:

$$\|\mathbf{B}\| < 1 \tag{3}$$

The norm of matrix $\mathbf{B}$, cf. Eqn. (3) can be small in some norms and large in others, therefore a more stringent condition based on spectral radius, ρ of matrix, $\mathbf{B}$ i.e. $\rho(\mathbf{B}) < 1$ [2] is used in our work.

3. Numerical example

A 2D rectangular sample of equal sized particles (cf. Fig 2) has been used to numerically verify the convergence criterion defined by Eqn. (3). The discrete model consisting of 180 bonded disks of radii $r = 1$ mm and normal contact stiffness $k_n = 7 \cdot 10^{10}$ N/m was simulated under unconfined uniaxial compression mode. It will be shown that the specific form of convergence criterion (cf. Eqn. (3)) for the rectangular sample is given as,

$$\frac{4k_n(1+\nu_p)}{\pi E_p l} < 1 \tag{4}$$

where, k_n is normal contact stiffness, ν_p is particle Poisson's ratio and E_p is particle Young's modulus. A substantial number of simulations have been done for the particle Poisson's ratio ν_p ranging between 0.05 to 0.45 and a comparison between numerical and analytical convergence limit is presented.

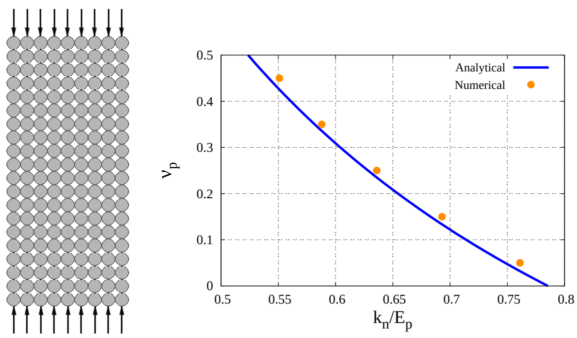

Figure 2: Verification of convergence criteria for an rectangular discrete sample of equal size particles

It can be seen from Fig. 2, that indeed we can predict the convergence limit of a DDEM model analytically and hence select the microscopic elastic parameters suitably to obtain a convergent solution using DDEM model.

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