A NOVEL TREATMENT FOR THE DEFORMABILITY OF DISCRETE ELEMENTS

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1. Introduction

Correct representation of macroscopic properties using standard DEM models is still a challenge. Even the so-called *soft-contact* [1] approach of contact treatment which allows a small particle overlap equivalent to particle deformation at contact is not always sufficient to obtain a required deformation behaviour [5]. Different approaches reported in the literature to incorporate particle deformability within DEM include the use of finite elements to discretize particles [4] and adding deformation modes to a rigid motion for polygonal or polyhedral elements [7]. Works considering the deformability of circular elements with soft contact approach are rather very few. For example, Haustein et al. [2] presented a simple approach purely based on geometrical assumptions. This paper presents a novel treatment for the deformability of the circular discrete elements consisting in adding a global deformation mode induced by an average stress tensor derived from the contact forces acting on each particle. An interesting feature of our novel method is that by using appropriate model parameters, we can recover the finite element solution of the problem. Our new formulation is termed as deformable discrete element method or DDEM and an overview of its formulation is presented further. For a detailed discussion on DDEM see [6].

2. Formulation of the novel concept for treating particle deformability

The idea of our novel concept for treating particle deformability is shown in Fig. 1. Under the uniform stress assumption, the internal particle stress is obtained from contact forces as volume average stress using the formula [3]:

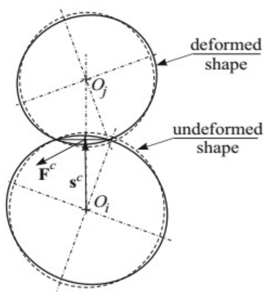


Figure 1: The idea of the deformable discrete element method

$$\tilde{\boldsymbol{\sigma}}_p = \frac{1}{V_p}\sum_{c=1}^{n_{pc}} \frac{1}{2}\left(\mathbf{s}^c \otimes \mathbf{F}^c + \mathbf{F}^c \otimes \mathbf{s}^{\mathbf{c}}\right) \tag{1}$$

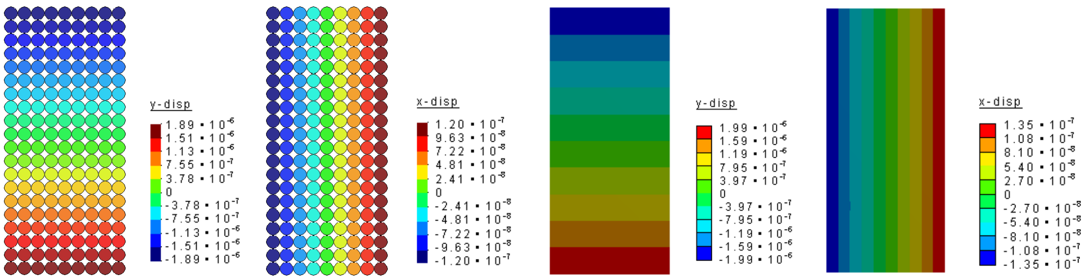
where V_p – particle volume, n_{pc} – no. of elements in contact, $\mathbf{s}^c$ is vector connecting particle center with contact point, $\mathbf{F}^c$ – contact force and symbol $\otimes$ – outer tensor product. An inverse constitutive relationship is used to obtain particles strains,

$$\epsilon_p = \mathbf{D} : \boldsymbol{\sigma}_\mathbf{p} \tag{2}$$

where $\mathbf{D}$ is elastic compliance tensor for plane strain. These strains elicit new contacts and concurrently lead to change in local particle overlap in our soft contact model due to deformation of a circular disk into an ellipse with axis aligned along principal strain directions (cf. Fig(1)). Hence, the deformability of discrete elements invokes interdependency of contacts, similar to that of a non-local contact model. This is the distinct characteristic of our new formulation as compared to standard DEM. It will be shown that in-fact we can retrieve a FEM solution from a DDEM model using suitable parameters. A numerical example will be presented for its verification.

3. Numerical example

The equivalence between finite element solution and deformable discrete element method is verified numerically by simulating uniaxial compression of a 2D rectangular sample. The DEM sample which is assumed to represent an elastic solid is discretized with 180 bonded disk elements of radii $r = 1$ mm. Equivalent FEM model has been obtained by taking porosity of the DEM sample into account. Figure 2 shows the displacement contours in x and y directions for DDEM model (a) and b)) and an equivalent FEM model (c) and d)). From the results presented in Fig. 2, it can be observed that in-fact, our novel method to treat deformability of discrete elements allows us to obtain a solution equivalent to the FEM one, and which captures correctly the Poisson's effect which in this case would be impossible with the standard DEM model.



a) b) c) d)

Figure 2: Displacement contours – simulation results obtained with: new DEM formulation – contours of displacements along a) the y-axis, b) the x-axis; equivalent FEM model – contours of displacements along c) the y-axis, d) the x-axis; [6]

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