ON THE EFFECTIVE PROPERTIES OF THE OCTET-TRUSS LATTICE

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1. Architectural Materials

In the last few decades there has been a growing interest in lightweight load-bearing structures. Inspiration from nature can be found in natural cellular materials like wood, honeycomb, butterfly wings and foam-like structures such as trabecular bones and sponge [1]. Architectural cellular materials have been used to create mechanically-efficient engineering structures such as the Eiffel Tower and the Garabit Viaduct. This class of materials combines the benefits of low density as it only occupies a fraction of the monolithic bulk solid, and strength by arranging its elements efficiently to carry the loads. Previous studies have shown that the macroscopic mechanical properties of cellular materials depend on three parameters: the constituent material properties, the deformation mechanism, and the relative density $\overline{\rho}$ (defined as the solid volume within the unit cell divided by the volume of the unit cell). Cellular-solids theory predicts scaling relationships between the macroscopic stiffness and strength vs. the relative density, namely $E_s \; \alpha \; \overline{\rho}^m$ and $\sigma_y \; \alpha \; \overline{\rho}^n$ respectively, where the dimensionless parameters m and n depend on the unit cell geometry [2].

For a 3D structure to be rigid (i.e. statically and kinematically determinate), a minimum nodal connectivity of $Z = 6$ is required. A connectivity of $Z = 12$ categorizes the structure as stretching-dominated where the lattice members deform by tension/compression. Bending-dominated structures that deform through the bending of their members, has a connectivity of $6 \leq Z < 12$ [3]. For stretching-dominated structures such as the octet-truss lattice, these scaling relationships are linear. On the other hand, for bending-dominated structures such as honeycombs or the octahedral lattice, they are quadratic or stronger [4].

When the dimensions of the lattice members are scaled down below the micron length scale, they exhibit different mechanical behaviour. Examples of these size-dependent changes include strengthening in single crystalline metals and transition from brittle to ductile behaviour in metallic glasses and ceramics [5], [6]. Recent advances in additive manufacturing techniques have made it possible to manufacture lattice structures with more geometrical and dimensional freedom. Certain AM techniques like self-propagating photopolymer waveguides [7], projection micro stereolithography [1], and two-photon lithography have been utilized to produce micro and nanolattices within the length scales required to activate material size effects. This is in addition to the structural effects activated by changing the various geometric parameters of the lattice unit cell [8].

2. Constitutive Modelling of the Octet-truss

Continuum constitutive models have been developed to describe the effective mechanical properties of the octet-truss lattice structure. A common assumption amongst these models is that the lattice members are pin-jointed at all nodes, hence the contribution from the bending resistance of the members and nodes can be neglected compared to the axial tensile/compressive stiffness of the members. Deshpande et al. (2001) checked the accuracy of the pin-jointed assumption by comparing FE calculations of rigid-jointed structures against analytical values of pin-jointed models for relative densities $\bar{\rho}$ ranging 0.01 to 0.5, the results showed excellent agreement between the FE and analytical values proving the validity of this assumption [3]. Generally, symmetry considerations could be employed to deduce the number of independent constants in the macroscopic stiffness tensor. Following the pin-jointed assumption, these elastic constants are determined by averaging the contribution from each element to the macroscopic stiffness, which is achieved through 3D coordinate transformations.

Nayfeh and Hefzy (1978) derived a first order approximation of the relative density of the octet-truss lattice by dividing the solid volume within the unit cell by the total volume of the unit cell. They employed 3D coordinate transformation and volume averaging in order to obtain the macroscopic stiffness matrix. Lake

(1992) constructed a strength tensor by converting applied stresses to strains for each parallel group of members using the macroscopic compliance matrix. Failure would occur in a member if its axial strain exceeded a critical value based on an elastic buckling limit. The choice of elastic buckling over plastic yielding is somehow justified given that space structures, the typical application of lattice structures at that time, usually compose of slender members. Lake's strength tensor could easily accommodate multiaxial loading as well as different loading directions through coordinate transformation. The author also developed a 3D plot of the uniaxial compression strength in cartesian coordinates, from which he concluded the direction and value of the maximum strength of the octet-truss lattice for the case of cubic symmetry where the lattice angle θ equals $45°$ (the angle between the individual members and the horizontal midplane). Deshpande et al. (2001) investigated the effective properties of the octet-truss lattice structure both theoretically and experimentally [3]. They validated the analytically-predicted elastic modulus and strength using FEM and experimental uniaxial compression of octet-truss lattice made from a casting aluminium alloy.

It is important to note that the previous studies were performed only for the case of cubic symmetry. At this angle, the octet-truss is considered to be at the highest attainable level of symmetry. However, potential applications of metamaterials (e.g. thin-walled pressure vessels) necessitates the use of anisotropic lattice structures in order to achieve the optimal combination of low density and high load-carrying capacity.

3. Methodology and Results

The purpose of the present research is to investigate the effect of the lattice angle on the effective properties of the octet-truss lattice structure, namely the effective stiffness. The appropriate steps are followed to develop a continuum-based analytical model of the octet-truss lattice while including the lattice angle parameter θ. The output of these analytical derivations are the stiffness/compliance tensors. Isotropic and homogenous properties are assumed for the constituent material. The pin-jointed nodes assumption is assumed to simplify the derivations, where we only consider the axial compressive/tensile stiffness of the truss members and ignored the nodes and members bending resistance. This assumption aligns with the stretching-dominated behaviour of the octet-truss. General expressions for the effective elastic moduli of the octet-truss for a general lattice angle are obtained using two consecutive stiffness tensor transformations. Tri-dimensional polar representations of effective the elastic modulus for different lattice angles show that the loading direction of the maximum elastic modulus always lies in a plane perpendicular to the $x - y$ plane at $\varphi = 45°$. As θ increases, this direction moves closer to the z axis. As θ decreases, it moves closer to the $x - y$ plane. A Lattice angles less than $45°$ produce higher overall effective specific elastic moduli, specifically in the $x - y$ plane. A plot of the maximum and minimum specific stiffness against the lattice angle describes the anisotropic behaviour of the octet-truss lattice.

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