AN EFFICIENT NONLOCAL MODEL FOR PRELIMINARY DESIGN OF SANDWICH RECTANGULAR PLATE WITH LAMINATED FACINGS

S. Karczmarzyk

Institute of Vehicles, Faculty of Automotive and Construction Machinery Engineering, Warsaw University of Technology, Warsaw, Poland

e-mail: karczmarzyk_st@poczta.onet.pl

1. Introduction

Layered surface girders (plates and shells) can be divided into two groups: laminate and sandwich. The sandwich constructions are usually composed of three layers, with different thicknesses h_k, $k = 1,2,3$, fulfilling the condition that the ratio h_2/h_k, $k = 1,3$, is much greater than one. Usually $h_2 / h_k \in (20-100)$. The second indicator characteristic of these sandwich constructions is the ratio of Young's modules of adjacent layers , ie E_k/E_2, $k = 1,3$. Usually $E_k / E_2 \in (100-10000)$. As it results from the above values of the ratios, the sandwich constructions are physically and geometrically inhomogeneous with abruptly variable parameters. If we take into account that the outer layers of the sandwich girders can be laminated, it is obvious that the exact elastic mathematical models of these girders are usually very complex.

The sandwich plates have been attractive to many researchers interested in the structural aspects of engineering constructions. Elastic models of sandwich plates can be divided into two groups: non-local and local. The local models are presented, for example, in the following papers [1-2]. As far as the author knows, all the previous local models are limited to rectangular simply supported plates. The non-local models are much more frequently published in the literature and therefore their exact classification is not very easy because of the large number of scientific papers devoted to them. Here, non-local models are divided into two groups: the equivalent single layer (ESL) models and individual-layer (I-L) models. Below there are mentioned some exemplary works containing the ESL and I-L models (theories). A simple ESL theory of sandwich plate one can find in [3]. In paper [4] an ESL refined theory for the sandwich plate with laminated facings was presented. Also in paper [5] an ESL refined theory has been presented in detail. In paper [6] an I - L theory of sandwich plate was outlined.

This presentation concerns an I-L model for the rectangular sandwich plate composed of laminated outer layers (facings) and an orthotropic middle layer (core). The plate is symmetric about the middle plane.

2. Outline of the present individual-layer model of sandwich plate with laminated facings

Displacements vector $\underline{u}_k$ occuring within k^{th} layer, $k = 1\text{-}3$, as well as the corresponding stresses within this model satisfy the compatibility equations between the adjacent layers, i.e.

(1) $$\underline{u}_1 = \underline{u}_2 \ , \qquad \underline{u}_1 = \underline{u}_2 \ , \qquad \underline{u}_k = \left[u_{xk} \ \ u_{yk} \ \ u_{zk} \right]^T ,$$

(2) $$\underline{\sigma}_1 = \underline{\sigma}_2 \ , \qquad \underline{\sigma}_1 = \underline{\sigma}_2 \ , \qquad \underline{\sigma}_k = \left[\sigma_{zx} \ \ \sigma_{zy} \ \ \sigma_{zz} \right]_k^T .$$

Of course, the cross-sectional boundary conditions, for the first and third layer, have also been satisfied. The local constitutive equations applicable in k^{th} layer are consistent with the kinematic model for the layer.

To obtain displacement vectors $\underline{u}_k$ the following equations, for each k^{th} layer separately, have been applied,

(3) $$\int_{z_{1k}}^{z_{2k}} \left(\frac{\partial (\sigma_{xx})_k}{\partial x} \right) z dz + \int_{z_{1k}}^{z_{2k}} \left(\frac{\partial (\sigma_{xy})_k}{\partial x} \right) z dz + \int_{z_{1k}}^{z_{2k}} \left(\frac{\partial (\sigma_{xz})_k}{\partial x} \right) z dz = \int_{z_{1k}}^{z_{2k}} \left(\frac{\partial^2 (u_x)_k}{\partial t^2} \right) z dz,$$

(4) $$\int_{z_{1k}}^{z_{2k}}\left(\frac{\partial(\sigma_{yx})_k}{\partial x}\right)zdz+\int_{z_{1k}}^{z_{2k}}\left(\frac{\partial(\sigma_{yy})_k}{\partial x}\right)zdz+\int_{z_{1k}}^{z_{2k}}\left(\frac{\partial(\sigma_{yz})_k}{\partial x}\right)zdz=\int_{z_{1k}}^{z_{2k}}\left(\frac{\partial^2(u_y)_k}{\partial t^2}\right)zdz,$$

(5) $$\int_{z_{1k}}^{z_{2k}}\left(\frac{\partial(\sigma_{zx})_k}{\partial x}\right)dz+\int_{z_{1k}}^{z_{2k}}\left(\frac{\partial(\sigma_{zy})_k}{\partial x}\right)dz+\int_{z_{1k}}^{z_{2k}}\left(\frac{\partial(\sigma_{zz})_k}{\partial x}\right)dz=\int_{z_{1k}}^{z_{2k}}\left(\frac{\partial^2(u_z)_k}{\partial t^2}\right)dz.$$

Finally, equations (3) are summed up for $k = 1 - 3$, ie through the entire thickness of the plate. The same applies to equations (4) and (5).These final equations, derived after the summations, make it possible to determine the functions of $\underline{u}_k$. To obtain complete statement of the boundary problem the edge boundary conditions have to be satisfied. For example, for a plate with fixed edges these conditions are as follows,

(6) $$\begin{aligned} x=0,a &\quad\Rightarrow\quad u_{zk}=0, \quad \frac{\partial u_{zk}}{\partial x}-\frac{T_{zx}}{S_{zx}}=0,\\ y=0,b &\quad\Rightarrow\quad u_{zk}=0, \quad \frac{\partial u_{zk}}{\partial y}-\frac{T_{yz}}{S_{yz}}=0. \end{aligned}$$

Symbols T_{yz}, T_{zx}, denote the transverse edge shear forces, S_{yz}, S_{zx}, are the shear stiffnesses of the plate, while symbols a, b denote dimensions of the plate.

3. Local model for cellular core

The middle layer (core) of the sandwich panel can be of continuous or cellular material. There are many “continuum” models in the literature for the cellular core which simplify modelling of the sandwich plate. For instance, in [7] the following formulas for the Young’s and shear modulus of the cellular core are given,

(7) $$E_c=E_s\left(\frac{\rho_c}{\rho_s}\right)^2, \qquad G_c=\frac{3}{8}E_s\left(\frac{\rho_c}{\rho_s}\right)^2.$$

Symbols E_c, G_c, ρ_c, in expressions (7), denote equivalent Young’s modulus, shear modulus and density, respectively, of the “isotropic” cellular core while E_s, G_s, ρ_s, are the same parameters of the solid material from which the core is made. It is noted that directly from (7) we have the Poisson’s ratio value, $v_c = 1/3$. It is noted that many other formulas analogous to (7) can be found in the literature.

The I-L model of a rectangular sandwich plate commented here is much simpler in comparison with other I - L models existing in the literature. Therefore, it is useful for the preliminary design of the plate eg to analyse the influence of type and arrangement of reinforcements in laminate layers on stiffness of the plate.

References

[1] S. Srinivas, A.K. Rao. Bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates. *Int. J. Solids & Struct.*, 6: 1463-1481, 1970.

[2] A.T. Jones. Exact natural frequencies and modal functions for a thick of-axis lamina. *J. Comp. Mat.*, 5: 504, 1971.

[3] D. Zenkert. *An introduction to sandwich construction*. EMAS Publishing, 1995.

[4] T. Kant, K. Swaminathan. Analytical solutions for the static analysis of laminated and composite sandwich plates based on a higher order plate theory. *Composite Structures*, 56: 329-732, 2002.

[5] M. A-A. Meziane, H. H. Abdelaziz and A. Tounsi. An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. *J. Sandwich Struct. and Materials*, 293-318, 2014.

[6] K. Malekzadeh and A. Sayyidmousavi. Free vibration analysis of sandwich plates with a uniformly distributed attached mass, flexible core, and different boundary conditions. *J. Sandwich Struct. and Materials*, 709-732, 2010.

[7] J. B. Min, L. J. Ghosen and B. A. Lerch. A study for stainless steel fan blade design with metal foam core. *Journal of Sandwich Struct. and Materials*, 56-73, 2015.