TRANSIENT THERMAL STRESSES IN A FUNCTIONALLY GRADED CYLINDER BY PSEUDOSPECTRAL CHEBYSHEV METHOD

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Introduction

Functionally graded material (FGM) can be categorized as advanced engineering material that is able to survive in a severe working environment by preserving its properties during service. They are designed with varying properties that include changing chemical, mechanical, magnetic, thermal, and electrical properties [1]. FGMs are used in many applications exposed to high temperature such as; energy conservation system, aerospace and nuclear energy applications, heat engine component. The temperature gradient on the material give rise to thermal stress due to different amount of expansions, thus transient stresses are the maximum thermal stress on the material. If the thermal shock is severe enough, crack initiation-propagation and creep may occur. Therefore, it is important to analyze the internal thermal stresses in FGMs and to evaluate their resistance to thermal loading such as thermal shock [2].

Transient thermoelastic analysis of a FG cylinder under thermal loading was studied. The cylinder material is considered to be graded along the radial direction, where an exponentially varying distribution is assumed. The cylinder is subjected to a constant temperature at the surface. The governing equation of the cylinder transformed into the Laplace space and then solved numerically by Chebyshev pseudospectral approach in radial direction for transient condition. Pseudospectral Chebyshev method is a global method and converges at a rate that is faster than that of conventional methods. It can be achieved a great accuracy for coarse grid points. Solutions were transformed from Laplace domain to the time domain by applying modified Durbin's procedure. The time dependent temperature, radial displacement and stress distributions are examined for a FG cylinder consists of ceramic ZrO_2 and alloy $Ti-6Al-4V$. The method is validated with the literature.

Basic Equations

Heat Conduction

The transient temperature distribution in a FG circular cylinder is considered under prescribed thermal boundary conditions. The material properties are assumed to change in radial direction. The heat conduction equation in axisymmetric cylindrical coordinates-to solve at first- without heat generation,

(1)
$$\frac{\partial^2 T}{\partial r^2}+\frac{\partial T}{\partial r}\left(\frac{1}{r}+\frac{1}{k(r)}\frac{dk}{dr}\right)=\frac{\rho(r)c_p(r)}{k(r)}\frac{\partial T}{\partial t}$$

where $T(r,t)$ is the time dependent temperature field and k, ρ, c_p and t are thermal conductivity, density, specific heat at constant pressure and time, respectively.

Thermal Stress

A long FGM cylinder of an isotropic material is investigated. In this case, the deformation of the cylinder is expressed through the radial stress σ_r, the hoop stress σ_ϕ, and the axial stress σ_z. So the thermoelastic stress-strain relations are given as follows:

$$\sigma_r = c_{11}\epsilon_r + c_{12}\epsilon_\phi - (c_{11} + 2c_{12})\alpha\Delta T$$
$$\sigma_\phi = c_{12}\epsilon_r + c_{11}\epsilon_\phi - (c_{11} + 2c_{12})\alpha\Delta T \tag{2}$$
$$\sigma_z = c_{12}(\epsilon_r + \epsilon_\phi) - (c_{11} + 2c_{12})\alpha\Delta T$$

where c_{ij} and α are the elastic constants and thermal expansion coefficient, respectively. $\Delta T = T - T_\infty$ is the temperature difference with the ambiant temperature T_∞. Equation of stress equilibrium is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\phi}{r} = \rho \frac{\partial^2 u}{\partial t^2} \tag{3}$$

By inserting elastic constant and strain components ($\epsilon_r = du/dr,\ \ \epsilon_\phi = u/r$) into Eqs. (2) and then into Eq. (3), the dynamic equation of motion for the displacement u can be obtained as

$$\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r}\left[\frac{1}{r} + \frac{1}{E}\frac{dE}{dr}\right] + \frac{u}{r}\left[\frac{\nu}{1-\nu}\frac{1}{E}\frac{dE}{dr} - \frac{1}{r}\right] = \frac{1+\nu}{1-\nu}\left[\frac{\partial}{\partial r}(\alpha\Delta T) + \alpha\Delta T \frac{1}{E}\frac{dE}{dr}\right] + \frac{(1+\nu)(1-2\nu)}{(1-\nu)}\frac{\rho}{E}\frac{\partial^2 u}{\partial t^2}. \tag{4}$$

When the solution of u is known from Eq. (4), the stresses and strains can be determined from Eqs. (2). Since the stresses at the center are expected to be to be finite, the displacement u must vanish at $r = 0$. And, the cylinder is assumed to be free of surface tractions at the outer surface.

Pseudospectral Chebyshev Model

Pseudospectral Chebyshev Model used to perform transient thermal stress analysis of FG cylinder by referring to the study of Gottlieb [3] that depends on discretization the governing equations (1-4) with respect to the radial variable using pseudospectral Chebyshev method. With regard to collocation points, the first order $(n+1) \times (n+1)$ Chebyshev differentiation matrix

$$0 = r_0 < r_1 \cdots < r_n,\ \ with\ \ r_j = \frac{1}{2}[1 - \cos(j\pi/n)] \tag{5}$$

$(j = 0, 1, \cdots, n)$ will be denoted by D. First-order Chebyshev differentiation matrix D provides highly precise approximation to $u^{'}(r_j), u^{''}(r_j), ...$, simply by multiplication differential matrix with vector data $u^{'}(r_j) = (D\ u)_j,\ u^{''}(r_j) = (D^2 u)_j$, suchlike where $u = [u_0, \ldots, u_n]^T$ discrete vector data at positions r_j.

The computation procedure of the Chebyshev differentiation matrix and codes as m-file can be found in notable references, see e.g., Trefethen [4], where the collocation points x_i are numbered from right to left and defined in $[-1, 1]$. With a small revision, differentiation matrix D can be implemented to the any interval.

References

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