STEADY VIBRATIONS PROBLEMS IN THE THEORY OF THERMOELASTICITY FOR MATERIALS WITH TRIPLE VOIDS

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1. Introduction

The prediction of the thermomechanical properties of materials with single and multiple porosity has been one of hot topics of continuum mechanics for more than one hundred years. The mathematical models for materials with single and multiple voids have found applications in many branches of civil and geotechnical engineering, technology and biomechanics. In this connection, Cowin and Nunziato [1] presented the linear theory of elastic materials with single voids. Ieşan [2] extended this theory and developed a linear theory of thermoelastic materials with single voids. Nowadays, these mathematical theories are extensively investigated by several authors and the basic results may be found in the books of Ciarletta and Ieşan [3], Ieşan [4], Straughan [5] and references therein.

Further, Ieşan and Quintanilla [6] developed the theory of thermoelastic materials with double voids by using the mechanics of materials with voids. Moreover, in this new theory the independent variables are the displacement vector, the volume fractions of pores and fissures and the variation of temperature. The basic boundary value problems (BVPs) of equilibrium and steady vibrations in the theory of elasticity for materials with double voids are investigated by using the potential method by several authors (see [7]- [10]). Most recent results in the theories of materials with double voids are given in the new book of Straughan [11].

Recently, Svanadze [12] presented the linear equilibrium theory of elasticity for materials with triple voids structure by using the concept of the mechanics of materials with voids, and the uniqueness and existence theorems for regular (classical) solutions of the internal and external BVPs of equilibrium are proved by using the potential method.

In this work the mathematical model of the linear thermoelasticity for materials with triple voids (macro-, meso- and microporosity) structure is developed. In this model the independent variables are the displacement vector, the volume fractions of macro-, meso- and micropore networks and the variation of temperature. The BVPs of steady vibrations of this theory are investigated by using the potential method.

2. Basic Equations

Let $\mathbf{x} = (x_1, x_2, x_3)$ be a point of the Euclidean three-dimensional space $\mathbb{R}^3$. In what follows we consider an isotropic and homogeneous elastic material with triple porosity structure that occupies a region of $\mathbb{R}^3$; $\mathbf{u}(\mathbf{x})$ denote the displacement vector, $\mathbf{u} = (u_1, u_2, u_3)$; $\varphi_1(\mathbf{x})$, $\varphi_2(\mathbf{x})$ and $\varphi_3(\mathbf{x})$ are the changes of volume fractions from the reference configuration corresponding to macro-, meso- and micropore networks, respectively; θ is the temperature measured from the constant absolute temperature T_0 ($T_0 > 0$).

Within a dual porosity conceptual framework [6], the system of homogeneous equations of steady vibrations in the linear theory of thermoelasticity for materials with triple voids structure has the following form

$$
\begin{aligned}
&\mu\Delta\mathbf{u} + (\lambda+\mu)\nabla\,\mathrm{div}\mathbf{u} + b_j\nabla\,\varphi_j - \gamma_0\nabla\theta + \rho\omega^2\mathbf{u} = \mathbf{0},\\
&(\alpha_{lj}\,\Delta + \rho_{lj}\omega^2 - a_{lj})\,\varphi_j - b_l\,\mathrm{div}\mathbf{u} + \gamma_l\theta = 0,\\
&k\Delta\theta + i\omega T_0(a\,\theta + \gamma_0\,\mathrm{div}\mathbf{u} + \gamma_j\varphi_j) = 0,
\end{aligned}
\tag{1}
$$

where Δ is the Laplacian operator, $l = 1, 2, 3$; ρ is the reference mass density, $\rho_{lj} = \delta_{lj}\rho_j$ (no sum), δ_{lj} is the Kronecker delta, ρ_1, ρ_2 and ρ_3 are the coefficients of the equilibrated inertia corresponding to macro-, meso- and

micropore networks, respectively, $\rho_j > 0$ $(j = 1, 2, 3)$; ω is the angular frequency; $a, a_{lj}, b_l, k, \alpha_{lj}, \gamma_0, \gamma_l, \lambda$ and μ are the constitutive coefficients and repeated indices are summed over the range (1,2,3). We assume that the internal energy of the isotropic and homogeneous thermoelastic materials with triple voids is positive definite.

3. Basic results

The following results in the considered theory are obtained:

(i) The governing field equations of the linear theory thermoelasticity for materials with triple voids is presented by using the mechanics of materials with multiple voids;

(ii) The fundamental solution of the system (1) is constructed explicitly by means of elementary functions and its basic properties are established;

(iii) The basic internal and external BVPs are formulated. Green's formulae and Sommerfeld-Kupradze type radiation conditions are established;

(iv) The uniqueness theorems for these BVPs are proved. The basic properties of the surface (single-layer and double-layer) and volume potentials and singular integral operators are established;

(v) The BVPs are reduced to the equivalent singular integral equations for which Fredholm's theorems are valid;

(vi) Finally, on the basis of the potential method and the theory of singular integral equations the existence theorems for the classical solutions of the BVPs of steady vibrations are proved.

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