DEVELOPMENT OF THE MULTISCALE FINITE ELEMENT METHOD FOR THE ANALYSIS OF ADVANCED MATERIALS

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1. Introduction

In recent years, we observe a significant development of new materials, particularly composites and metamaterial, as well as the technological advancements that these new materials have allowed. To reduce the cost of experimental testing during the design stage of those materials, digital modeling can be incorporated. In order to represent correctly the materials' global mechanical behavior, it is necessary to include certain features of the microscale when modeling. However, the complexity of the problem leads to a large number of degree of freedom (dof). Therefore, we propose an application of the Multiscale Finite Element Method (MsFEM) [1] to reduce the computational cost.

The MsFEM is a promising method used to model heterogeneous materials. It requires neither the assumption of scale separation nor the periodicity of the microstructure. Furthermore, the calculations can be easily parallelized, since the MsFEM special macroscale shape functions that capture microscale details are computed independently, in appropriate groups of elements.

The proposed multiscale modeling will contribute to development of efficient and reliable tools for a better understanding of the relationship between the additive manufacturing process, metamaterial microstructure, and the overall mechanical properties of 3D printed elements.

2. Problem formulation

We consider the linear elasticity problems formulated below for heterogeneous or porous materials.

Find field of displacements u(x) in a domain Ω such that:

(1)
$$-\frac{\partial}{\partial x_j}\left(C^{\epsilon}_{ijkl}\frac{\partial u_k}{\partial x_l}\right) = f_i \qquad \forall \omega_s \subset \Omega$$

with kinematic ($\hat{u}$) and static ($\hat{t}$) boundary conditions specified on $\partial\Omega_D$ and $\partial\Omega_N$ respectively, where $\partial\Omega_D \cup \partial\Omega_N = \partial\Omega, \partial\Omega_D \cap \partial\Omega_N = \emptyset$ and continuity conditions at the interface between subdomains ω_s. The strong ellipticity and boundedness of the microscale material parameter tensor C^{ϵ} is assumed (i.e., $\exists \alpha, \beta \in R^+ : \alpha\xi_{ij}\xi_{ij} \le C^{\epsilon}_{ijkl}\xi_{ij}\xi_{kl} \le \beta\xi_{ij}\xi_{ij}, \quad \forall\, \xi_{ij} \in R^2$). The material parameters are differentiable (typically constant) in each ω_s, and f_i denotes a body force component.

The discrete counterpart of problem (1) may be written in the following matrix form

(2)
$$\boldsymbol{K}^h \boldsymbol{u}^h = \boldsymbol{f}^h$$

where $\boldsymbol{u}^h$ is the vector dof and $\boldsymbol{K}^h$, $\boldsymbol{f}^h$ denote the assembled matrix and vector, respectively.

The MsFEM uses two meshes of finite elements. A coarse mesh is generated at the macroscale and is locally refined, for each coarse element node. This creates a set of fine meshes for every element (see example in Fig. 1). The most important issue of the MsFEM is an approximation of solutions of the following auxiliary problems to determine special shape functions

Given Ψ_m, find Φ_m such that:

$$
\begin{cases}
\dfrac{\partial}{\partial x_j} C_{ijkl} \dfrac{\partial (\Phi_m)_k}{\partial x_l} = \dfrac{\partial}{\partial x_j} C^0_{ijkl} \dfrac{\partial (\Psi_m)_k}{\partial x_l} & \forall\, i = 1{,}2 \quad x \in L_H \\
\Phi_m = \hat{\varphi}_m & \text{on } \partial L_H
\end{cases}
\tag{3}
$$

In other words, problem (3) defines the interpolation operator that transfers M coarse element dof into N fine mesh dof. Such a mapping is represented by a matrix $I_{N\times M}$ and is used to compute the coarse element matrix $\boldsymbol{K}^H$, vector $\boldsymbol{f}^H$ of fine mesh.

$$
\boldsymbol{K}^H = \boldsymbol{I}^T \boldsymbol{K}^h \boldsymbol{I}, \qquad \boldsymbol{f}^H = \boldsymbol{I}^T \boldsymbol{f}^h \,. \tag{2}
$$

Assembling such a matrixes and vectors leads to a system of macroscale algebraic equations with relatively small number of dof.

3. Preliminary results

Below, we present a study of convergence of the method for on exemplary 2D problem (see Fig. 1).

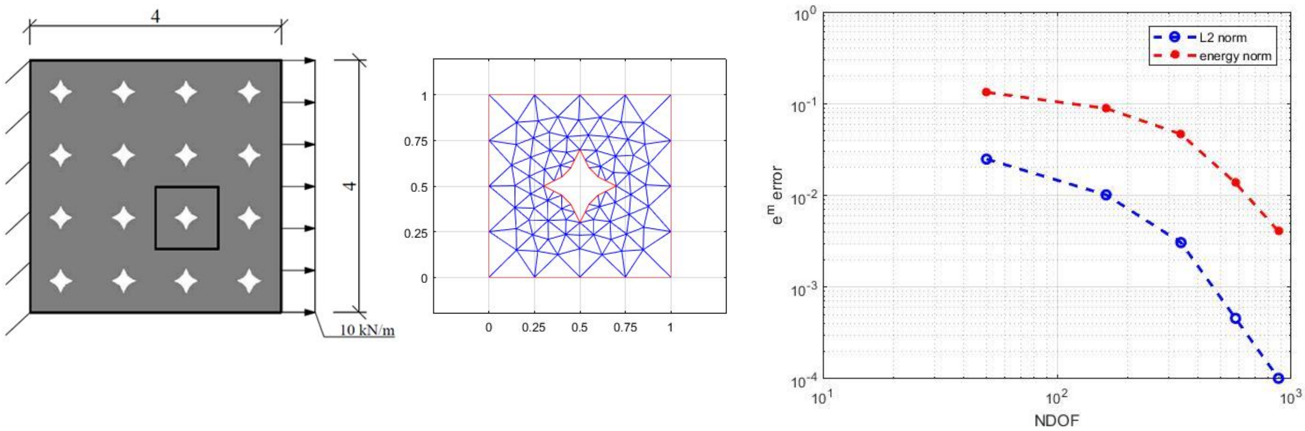

Fig. 1: Numerical example: the problem scheme and 4 × 4 element coarse mesh (left), fine mesh used for one coarse element (center), p-convergence of the error in L_2 and energy norms (right).

4. Concluding remarks

The results presented in the previous section as well as some other examples known from both literature and other research [2] indicate that MsFEM with higher order application is an efficient method for modeling of advanced materials. An experimental validation of the approach is conducted and its results will be discussed at the conference.

References

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