NOVEL ASPECTS IN DISLOCATION CONTINUUM THEORY: J-, M-, AND L-INTEGRALS

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1. Introduction

In this work, new aspects in dislocation continuum theory concerning the **J**-, M-, and **L**-integrals are presented within the framework of three-dimensional, linear, incompatible elasticity theory. First, the **J**-, M-, and **L**-integrals are derived for two straight (edge and screw) dislocations and second for a single (edge and screw) dislocation in isotropic materials. The results provide to the **J**-, M-, and **L**-integrals an important physical interpretation revealing their significance in dislocation continuum theory.

2. J-, M-, and L_3-integrals of straight dislocations

For two parallel edge dislocations with Burgers vectors in x-direction, the **J**-, M-, and L_3-integrals per unit dislocation length are given by [1]

(1) $$\frac{J_1}{l_z} = 2K^e_{xx}\,\frac{\cos\varphi\cos 2\varphi}{\bar r}\,,$$

(2) $$\frac{J_2}{l_z} = 2K^e_{xx}\,\frac{\sin\varphi(2+\cos 2\varphi)}{\bar r}\,,$$

(3) $$\frac{M}{l_z} = K^e_{xx}\left[2-\ln\frac{\bar r}{L}-\sin^2\varphi\right],$$

(4) $$\frac{L_3}{l_z} = K^e_{xx}\,\sin 2\varphi\,,$$

where

(5) $$K^e_{xx}(b_x, b'_x) = \frac{\mu b_x b'_x}{4\pi(1-\nu)}$$

is the *pre-logarithmic energy factor* for edge dislocations with Burgers vectors in x-direction. Here, μ is the shear modulus, ν is the Poisson ratio, L is the size of the dislocated body (or outer cut-off radius), $\bar r = \sqrt{\bar x^2+\bar y^2}$ is the distance between the two dislocations, and φ is the location angle of the dislocation with Burgers vector $\mathbf{b}$.

The M-integral between two edge dislocations with Burgers vectors in x-direction can be written in terms of the corresponding interaction energy as follows

(6) $$\frac{M}{l_z} = 2K^e_{xx} + \frac{1}{2}\frac{U_{\text{int}}}{l_z}\,,$$

where

(7) $$\frac{U_{\text{int}}}{l_z} = -2K^e_{xx}\left[\ln\frac{\bar r}{L}+\sin^2\varphi\right].$$

Eq. (6) states that the *M-integral of two parallel edge dislocations with Burgers vectors in x-direction per unit* *dislocation length is half the interaction energy between the two dislocations per unit length, plus twice the pre-logarithmic energy factor* K^e_{xx}.

Important results are summarized as follows:

- The **J**-integral of dislocations is the Peach-Koehler force (interaction force) between two dislocations.
- Eq. (6) provides to the M-integral the physical interpretation of the interaction energy between the two straight dislocations.
- The configurational work produced by the Peach-Koehler force for straight dislocations (per unit dislocation length) is constant, and equals twice the corresponding pre-logarithmic energy factor.
- The L_3-integral of two straight dislocations is the z-component of the configurational vector moment or the rotational moment (torque) about the z-axis caused by the interaction of the two dislocations.
- Fundamental relations between the **J**-, L_3-, and M-integrals of straight dislocations have been found and they show that the **J**-, L_3-, and M-integrals are not independent. If the M-integral is given, then the J_1-, J_2-, J_r-, J_φ-, and L_3-integrals can be easily calculated from it. From that point of view, the M-integral is of primary importance.
- The translational energy-release $\mathcal{G}^T_k$ of straight dislocations is identical to the J_k-integral.
- The rotational energy-release $\mathcal{G}^R$ of straight dislocations equals twice the value of the L_3-integral.

3. J-, M-, and L_3-integrals of a single dislocation

For a single edge dislocation with Burgers vector b_x, the **J**-, M-, and L_3-integrals per unit dislocation length, are given respectively [2]

$$\frac{J_1}{l_z} = 2K^e_{xx}\,\frac{1}{\epsilon}\,, \qquad \frac{J_2}{l_z} = 0\,, \tag{8}$$

$$\frac{M}{l_z} = K^e_{xx}\Big[\ln\frac{L}{\epsilon} + 2\Big]\,, \qquad \frac{L_3}{l_z} = 0\,, \tag{9}$$

where

$$K^e_{xx}(b_x) = \frac{\mu b_x^2}{4\pi(1-\nu)} \tag{10}$$

is the *pre-logarithmic energy factor* for a single edge dislocation with Burgers vector in x-direction. Here, ϵ is the inner cut-off radius being proportional to the constant dislocation core radius.

An important outcome is that the *M-integral (per unit length) of a single dislocation represents the total energy U_{total} of the dislocation (per unit length) which consists of the self-energy (per unit length) plus the dislocation core energy (per unit length)*

$$M/l_z = U_{\text{s}}/l_z + U_{\text{core}}/l_z = U_{\text{total}}/l_z\,. \tag{11}$$

The dislocation core energy can be identified with the work done by the Peach-Koehler force. It is shown that the dislocation core energy (per unit length) is twice the corresponding pre-logarithmic energy factor.

References

[1] E. Agiasofitou and M. Lazar. Micromechanics of dislocations in solids: J-, M-, and L-integrals and their fundamental relations, *Int. J. Eng. Sci.*, 114, 16-40, 2017.

[2] M. Lazar and E. Agiasofitou. Eshelbian dislocation mechanics: J-, M-, and L-integrals of straight dislocations, *Mech. Res. Commun. (Special Issue G.A. Maugin)*, http://dx.doi.org/10.1016/j.mechrescomm.2017.09.001, 2017.