INCLUSION SHAPE IN MEAN-FIELD MICROMECHANICAL MODELS

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1. Introduction

The shape of inhomogeneity in micromechanical approach, which uses an inclusion-matrix concept, is modelled by the concentration tensor. The tensor relates a local average of mechanical field with its macroscopic counterpart. The concentration tensors for the ellipsoidal inclusion shape are well examined in the literature. In [1], the authors have compared several strategies for non-ellipsoidal heterogeneity in the context of homogenization methods: analytical based on Eshelby's tensor decomposition into isotropic and anisotropic part, and the semi-analytical Mori-Tanaka method with replacement tensor approach (RMTM).

Incorporation of additional microstructural parameters such as particle packing, size of inclusions or properties of the area between phases (interphase), with simultaneous sustainment of simple formulations, is possible using the Morphology-based Representative Pattern (MRP) approach [2]. The MRP concept is based on the idea of the composite sphere introduced by Hashin in 1962. The MRP is formulated for an elastic and isotropic continuous matrix and isotropically dispersed spherical inclusions with equal radiuses.

The MRP takes into account the mean minimum distance between inclusions $\bar{\lambda}$ and, by introducing an additional phase t_{int} at the boundary of the composite components, the effect of the particle size. In the simplest example of two-pattern approach the microstructure of composite is simplified using two representative patterns (see Fig.1): the first one governed by the n-phase GSC scheme (4-GSC), and the second one described by the classical self-consistent scheme (SC). The first n-GSC pattern can be changed to e.g. the RMTM pattern in order to study influence of inclusion shape. To account for the size effect in the first pattern an interphase is introduced, having a thickness t_{int} independent of the particle radius R_i and different properties than the basic two phases. To describe the packing effect, the coating thickness in this pattern is specified by half the mean minimum distance between nearest-neighbour particles $\bar{\lambda}/2$ in the RVE.
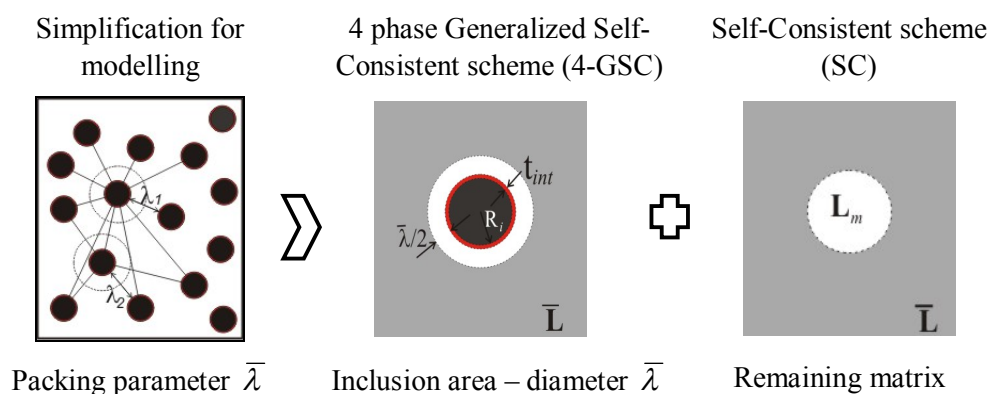

Simplification for modelling	4 phase Generalized Self-Consistent scheme (4-GSC)	Self-Consistent scheme (SC)
Packing parameter $\bar{\lambda}$	Inclusion area – diameter $\bar{\lambda}$	Remaining matrix

Fig.1 The idea of MRP approach. For the presented simplest case of MRP the mean radius R_i and the mean minimum distance $\bar{\lambda}$ between nearest-neighbour particles are calculated. The MRP is divided into two sub-problems: 4-GSC and SC, joint solution of these two sub-problems is final result of MRP model. GSC pattern describes an influence of local mechanical fields in the area of inclusion (spherical region of radius equal to $\bar{\lambda}/2$) on the macroscopic composite response and SC considers remaining matrix. The interphase, of different material parameter and thickness t_{int}, describes the size effect of inclusions.

2. Results

Analytical multi-scale models based on the inclusion-matrix concept, like MRP, take into account the shape of heterogeneity through a concentration tensor. Nogales and Böhm (2008) have extended the standard Mori-Tanaka scheme to inhomogeneities of non-ellipsoidal shapes by introducing the dilute "replacement" inhomogeneity strain concentration tensor $\mathbf{A}_{inc}^{FEM}$. For Mori-Tanaka method with replacement tensor approach (RMTM) the strain concentration tensor is

$$\mathbf{A}_{inc}^{RMTM} = [c_{inc}\mathbf{I} + c_m (\mathbf{A}_{inc}^{FEM})^{-1}]^{-1}$$

where $\mathbf{A}_{inc}^{FEM}$ for non-elliptical or non-ellipsoidal inhomogeneities has to be obtained numerically, e.g., from Finite Element (FE) simulations of a single inhomogeneity of appropriate shape and properties embedded into a large, but finite matrix region, see Fig.2b-d. The unit cells, Fig.2b-d, were exploited to determine the numerical concentration tensors for different inclusion shapes. The unit cells based on the microstructure photos obtained using Scanning Electron Microscope (SEM) and were modelled using FEM. The local stress and strain fields were computed at the Gauss points of FE for known macroscopic boundary conditions. Preliminary results concerning the effective Young modulus of polymer matrix composite reinforced with glass particles of different shape of inclusions are shown in Fig.2a. The numerical results are computed using numerical homogenization and FEM, the experimental results are adopted from [Kushvaha et al., 2014]. As the reinforcement volume increases, the stiffness of the composite increases, and the influence of the reinforcement shape is more visible.
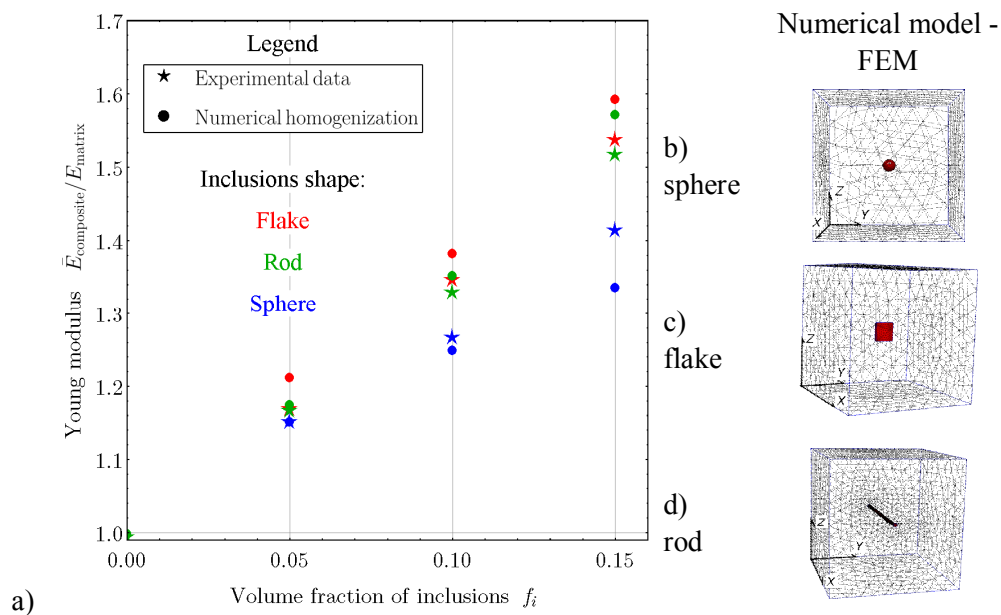

Fig.2 a) the macroscopic Young's modulus (composite/matrix) E_c/E_m vs. the volume fraction of inclusions f_i. The results of mechanical tests [Kushvaha et al., 2014] of polymer matrix composite reinforced with glass particles of various shapes: flake, rod and spherical is compared with the outcomes of numerical homogenization outcomes. b)-d) The unit cells set in FEM based on the microstructure SEM photos [Kushvaha et al., 2014].

References

[1] B. Klusemann, H.J. Bhom, and B. Svendsen, Homogenization methods for multi-phase elastic composites with non-elliptical reinforcements: Comparisons and benchmarks, *Eur. J. Mech. A/Solids,* 34:21–37, 2012.
[2] M. Majewski, M. Kursa, P. Holobut and K. Kowalczyk-Gajewska, Micromechanical and numerical analysis of packing and size effects in elastic particulate composites, *Compos. B Eng.,* 124:158-174, 2017.