INITIAL BOUNDARY VALUE PROBLEM IN THE FRAMEWORK OF FRACTIONAL VISCOPLASTICITY

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1. Introduction

The desirable goal of the consitutive modelling is to obtain a model that with a few material parameters is able to provide a good compatibility between the experimental and numerical results. Application of the fractional calculus to Perzyna type viscoplasticity presents a new approach for describing the inelastic behaviour of various materials. This may be useful especially in the case of materials that display rate-dependent properties and were subjected to dynamic loading. Fractional derivative introduces a new material parameters that comes from the non local character of this generalization. In several papers [3], [4], [5], [6] the fractional approach to classical plasticity was presented demonstrating a induced anisotropy and non-associated plastic flow.

2. Experimental background

The plastic deformation that leads to formation of instability bands in metal plates was discussed in [1]. Authors shown that in the process of achieving the maximum true stress one can observe a diffuse necking in the region where intersecting instability bands appear. As the deformation proceeds, the necking and the elongation of material concentrate in one of the bands (cf. Fig. 1) and in their area the rate of deformation increases. At this stage practically all of the plastic deformation is taking place within this band.

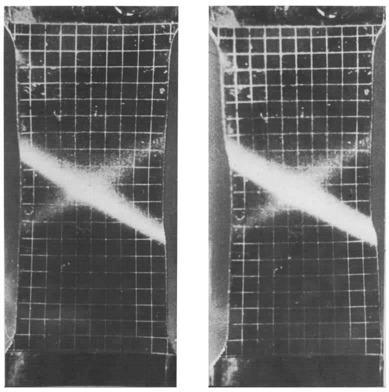

Figure 1: Development of instability bands and concentration of plastic deformation in predominant direction [1].

3. Fractional material model

The Theory of Thermo-Viscoplasticity was originally published by Perzyna in [2]. In the classical model the rate of inelastic strain is defined by a partial derivative of yield function F in the stress space. In the framework of fractional calculus this relation is reformulated to

$$\dot{\varepsilon}^p = \Lambda \mathbf{p} = \Lambda \, D^\alpha F \, ||D^\alpha F||^{-1}, \tag{1}$$

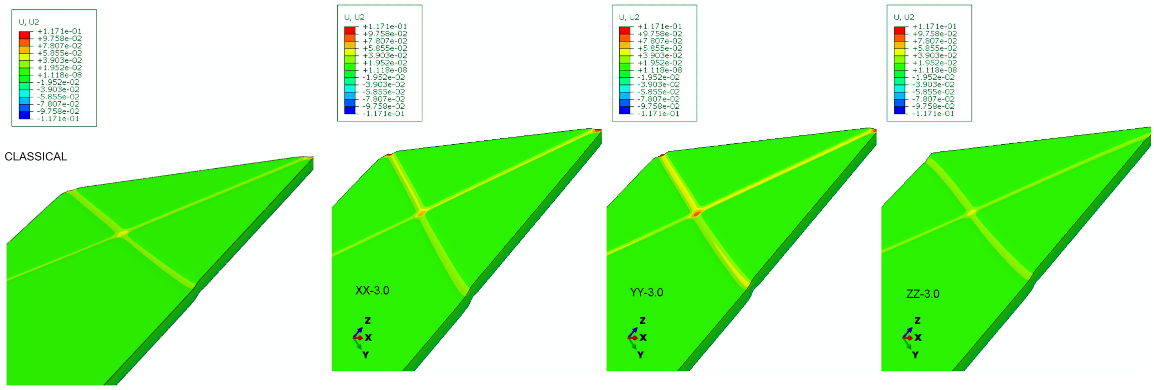

Figure 2: Necking achieved in numerical model for classical (leftmost) and fractional plasticity with different material parameters.

where Λ is a scalar multiplier and $D^{\alpha}F$ denotes the fractional operator. For the purpose of this paper the definition of the Riesz - Caputo (RC) derivative consisting of the left- and right-sided Caputo derivative is assumed

$$ {}_{a}^{RC}D_{b}^{\alpha} f\left(t\right) = \frac{1}{2}\left({}_{a}^{C}D_{t}^{\alpha} f\left(t\right) + \left(-1\right)^{n}\, {}_{t}^{C}D_{b}^{\alpha} f\left(t\right)\right), \tag{2} $$

where α denotes the order of the derivative $D(.)$ and whereas a, t, b are terminals describing the range on non-locality. Due to the limited number of analytical solution utilising fractional derivatives the Huber–Mises–Hencky (HMH) criterion was herein selected. The application of fractional operator results in new material parameters, namely the order of the derivative α and two six-dimensional vectors $\Delta^{L}_{(ij)}$ and $\Delta^{R}_{(ij)}$ that denotes the virtual neighborhood of a stress state. One of the advantages of this model is that it is possible to achieve the volume change, without any additional assumption, only through the modification of the new parameters. The built-in numerical VUMAT procedure for Abaqus/Explicit was used for numerical study of the aforementioned model. Simulations were conducted for a three-dimensional plates fixed at the one end while the other one was subjected to dynamic loading.

4. Results

The results were obtained for different velocities of the applied tension as well as for various material parameters α and Δ - Fig. 2. This type of examination allows to recognize how changes in parameters affect the localised viscoplastic deformation and exemplify the intensity of observed phenomena. This can be useful in the model's calibration process for a specific material. In future, a task of comparative analysis between experimental and numerical data will be considered.

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