MODELLING OF SEA-ICE PACK THERMODYNAMICS
BY THE SMOOTHED PARTICLE HYDRODYNAMICS METHOD

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1. Introduction

A typical sea-ice pack is a complex thermodynamic system comprising a multitude of floes of different size and geometry, driven by wind and water drag stresses, and subject to surface and basal freezing and melting in response to current local mechanical and thermal forcing. As individual floes move about and interact, in either ductile or brittle manner, they break, merge and override one another, giving rise to large variations in local ice thickness and ice area fraction (concentration). Since broken ice cover cannot carry tensile stresses, the mechanical behaviour of an ice pack in converging flow is remarkably different from that in diverging flow. The consequence of this is the development and subsequent propagation of interfaces that separate converging and diverging regions in sea ice, often leading to the fragmentation of an initially coherent pack domain. An important feature is also a significant change of the planar geometry of a domain occupied by the ice pack, associated with large displacements of boundaries between the coherent ice and the open sea. All these physical mechanisms are difficult to treat both mathematically and numerically, and significantly increase the complexity of numerical algorithms and the cost of calculations.

The equations describing the thermodynamic behaviour of sea ice have been derived by treating the problem as a two-dimensional on the 'horizontal' surface (the free surface of the ocean), and by integrating the mass, momentum and energy balances of the ice and lead water through the ice pack thickness (Gray and Morland [1]). The fundamental variables involved in the description are the local mean ice thickness, the ice concentration and two components of the velocity field. Sea ice is commonly treated as a viscous-plastic material (Hibler [2]), and its rheology is described by a constitutive law that relates the depth-integrated stresses to the two-dimensional deformation-rate. Since the strength of ice in tension is zero, the behaviour of an ice pack dramatically depends on whether the flow is locally converging or diverging. As a consequence, the structure of equations changes across interfaces separating converging and diverging flow regions, which can give rise to instabilities (Schulkes et al. [3], Guba et al. [4]) when solving the sea-ice flow equations by a numerical method.

In the present work, the sea-ice flow problem is solved by applying a mesh-free approach known as the Smoothed Particle Hydrodynamics (SPH). The SPH method is used to construct a numerical model which is applied to simulate the evolution of a coherent ice pack driven by wind stresses and subject to the mechanisms of ice grow or decay due to the phase changes (freezing of water and melting of ice). The thermodynamic processes resulting in the changes in ice mass are modelled in a simplified manner, by expressing the ice growth-rates by means of a single function of two arguments: local air temperature and current ice thickness. The latter function approximates the sea ice behaviour observed during a typical Arctic winter [2]. The ice is treated as a viscous-plastic material, and its rheology is described by a viscous fluid flow law, with two viscosity parameters bounded by an elliptic yield curve [2].

2. Smoothed particle hydrodynamics method

The Smoothed Particle Hydrodynamics (SPH) method was invented in 1977 by Lucy [5] and Gingold and Monaghan [6], but for nearly two subsequent decades its use was solely restricted to the field of astrophysics. Only in the mid 1990s some attractive features of the SPH method brought attention of the solid mechanics community, and ever since the interest in the method has been steadily growing and it has found applications in many branches of physics, applied mechanics and engineering, see Monaghan [7, 8]. The SPH method is fully Lagrangian and mesh-free, and owing to the fact that no connectivity between the particles is needed, the method has a natural capability of dealing with problems in which large deformations occur and surfaces of material discontinuity develop and subsequently propagate through the medium. Therefore, it seems that

the SPH approach is particularly well suited to solving the sea-ice pack flow problems, in which the application of conventional mesh-based discrete methods (such as the finite-difference or finite-element techniques, see Morland and Staroszczyk [9]) is difficult or entails significant numerical problems.

3. Sea-ice pack flow simulations

The SPH model has been implemented to simulate the evolution of a large sea-ice pack (of horizontal dimensions measured in tens or hundreds of kilometres) subject to the action of wind. First, the thermodynamic effects are ignored, and the model predictions for the viscous-plastic ice rheology are compared with those previously obtained for a non-linearly viscous rheology (Staroszczyk [10]), for an ice field under the action of wind of constant speed and direction. By the analogy with previously investigated problems solved by the finite-element method [3,9], an idealized, initially rectangular, geometry of the ice pack has been adopted, with two or three adjacent sides of the rectangle at solid boundaries (representing sea coasts), and the remaining two or one side(s) at open sea boundary. Then, the thermodynamic effects are included in the simulations, and a flow problem of an ice pack, initially of a uniform thickness and ice concentration, is solved. The pack is driven by a vortex geostrophic wind field, acting over at least several days, with the wind vortex centre located at the open sea off the initially rectangular ice cover. It seems that such a boundary value problem contains all the essential features which occur in realistic sea ice flows, and thus can serve as a test case for assessing the stability and performance of the applied mesh-free discrete model. The results of simulations illustrate the effects of the two rheological theories on the evolution of the ice pack, including the variations of the ice thickness and the ice area fraction in space and time. Further, the effects of different boundary conditions (free-slip and no-slip) assumed at the coast–ice interface are explored. Of particular interest are the changes in the position of an open sea boundary, as the tracking of the coherent sea-ice pack extent under given weather conditions is of practical importance to the navigation and the oil industry in the Arctic.

4. References

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