APPLICATION OF THE MESHLESS MONTE CARLO METHOD WITH RANDOM WALK PROCEDURE TO SELECTED ELLIPTIC PROBLEMS OF MECHANICS

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1. Introduction

This paper is focused on the development of the meshless Monte Carlo (MC) method with the random walk (RW) technique and its application to selected elliptic problems in mechanics. MC belongs to the wide class of probabilistic approaches and is commonly used in a variety of algebraic and differential problems. Its main concept is based upon the performance of series of simulations (trials) with properly defined success trial. Eventually, the number of success trials related to the total number of all trials, scaled by the dimension quantity (area, function value) may be treated as an unbiased estimator of the unknown solution to the considered problem. In this manner, MC combined with the RW technique, allows for a simple and effective estimation of the solution of the Laplace differential equation at selected point(s) of the domain, without a generation of a system of equation, combining all unknown function values [1]. However, the standard MC/RW approach is limited to an analysis of very simple elliptic problems, with essential boundary conditions and a regular mesh of nodes. In order to overcome those drawbacks, the original concept, proposed in this paper, is based upon the application of selected aspects of the meshless finite difference method (MFDM, [2]) to the MC/RW approach. Especially, the classification criteria of nodes into the FD stars as well as the local moving weighted least squares (MWLS, [2, 3]) approximation are taken into account. Therefore, an analysis of a wider class of problems, with more complex geometry, natural boundary conditions, non-homogeneous material and right-hand side functions as well as arbitrarily irregular clouds of nodes, is possible. The proposed meshless MC/RW approach is examined on selected 2D benchmark elliptic problems (torsion of a prismatic bar, stationary heat problem).

2. Standard Monte Carlo with random walk

When the Laplace equation is considered ($\nabla^2 F = 0$ in Ω with $F = \bar{F}$ on $\partial\Omega$), the following solution strategy may be adopted: 1. Select any arbitrary internal point $\mathbf{x}_k$ with unknown function value F_k, which belongs to the regular mesh of nodes (Fig.1a). 2. Randomly choose one of the four equally possible directions (north, east, south, west) and move to the closest node located there. 3. Proceed as long as the first boundary node is reached - note its function value $\bar{F}_i$ and increase its number of hits N_i (equal zero by default) by one. 4. Return to the point of interest $\mathbf{x}_k$ and repeat the entire procedure (1-3) until all N random walks are performed. 5. Estimate the function value F_k as

$$F_k \approx \frac{1}{N} \sum_{i \in \partial\Omega} \bar{F}_i N_i \qquad (1)$$

The mathematical proof [1] for the convergence of (1) to the true solution of Laplace problem is based upon the similarity of the stochastic system of equations to the one obtained by the standard finite difference method (FDM).

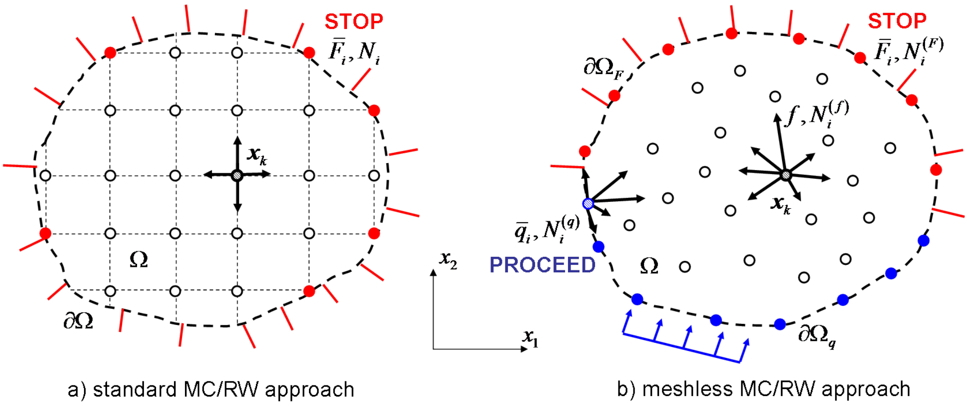

Figure 1: Comparison of the standard MC/RW (a) and the meshless MC/RW (b) concepts

3. Meshless Monte Carlo with random walk

More general elliptic equation is considered ($a\frac{\partial^2 F}{\partial x_1^2} + \frac{\partial^2 F}{\partial x_2^2} = f$ in Ω with $F = \bar{F}$ on $\partial\Omega_F$ and $\frac{\partial F}{\partial n} = \bar{q}$ on $\partial\Omega_q$). Moreover, if the domain Ω has a complex shape, an arbitrary irregular cloud of nodes has to be generated (Fig.1b). In such case, a directions of possible moves from the selected point $\mathbf{x}_k$ may be determined using meshless approach (e.g., FD stars with distance or cross criteria, [2]). Furthermore, selection probabilities depend on the distance between nodes in a FD star and the material function a value. They are found by means of the local MWLS technique [3], taking advantage from the information overload. The random walk terminates when the first boundary node $i \in \partial\Omega_F$ is reached. However, it continues for all internal and boundary nodes $i \in \partial\Omega_q$. The final meshless MC/RW formula combines all a-priori known information ($a, \bar{F}, f, \bar{q}$) as well as numbers of nodal hits N_i^F, N_i^f, N_i^q, determined in a stochastic manner, according to the MC concept.

4. Numerical examples

A variety of problems has been solved by the proposed MC/RW approach. Both regular meshes and strongly irregular clouds of nodes are investigated. Accuracy of solution and solution derivatives is taken into account. Solution convergence is terms of random walk number is examined. The most interesting up-to-date examples will be presented during the Conference.

5. Final remarks

Though obtained results are very promising, the method is still under current development. 2D+time (non-stationary), 3D as well as non-linear problems are planned as the following research stage. Additionally, the optimal balance between the number of walks N, and the number of nodes n needs to be determined.

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