THEORY AND FINITE ELEMENT FORMULATIONS FOR LAYERED COMPOSITE SHELLS

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1. Introduction

Shell elements which account for the layer sequence of a laminated structure are able to accurately predict the deformation behaviour of the reference surface, a sufficiently refined mesh presupposed. This holds also for the shape of the in–plane stresses, if the shell is not too thick. In contrast to that only averaged transverse shear strains through the thickness are obtained within the Reissner-Mindlin theory. As a consequence only the average of the transverse shear stresses is accurate. Neither the shape of the stresses is correct nor the boundary conditions at the outer surfaces are fulfilled. In a standard version the thickness normal stresses are neglected.

In several papers the equilibrium equations are exploited within a post–processing procedure to obtain the interlaminar stresses. The essential restriction of the approach is the fact that these stresses are not embedded in the variational formulation and an extension to geometrical and physical nonlinearity is not possible.

Higher order plate and shell formulations and layerwise approaches represent a wide class of advanced models. These theories are associated with global layerwise degrees of freedom which makes the general handling complicated for practical problems, e.g. when structures with intersections occur [1].

The use of brick elements or so-called solid shell elements represents a computationally expensive approach. For a sufficient accurate evaluation of the interlaminar stresses each layer must be discretized with several elements ($\approx 4 - 10$) in thickness direction. Especially for nonlinear large scale problems with a multiplicity of load steps and several iterations in each load step this is not a feasible approach [2].

An alternative to fully 3D computations is a multiscale formulation applying the so-called FE^2 approach. At the integration points of the macro scale representative volume elements (RVEs) are introduced. In case of thin structures the RVEs usually extend through the total thickness of the shell [3]. In comparison to one-scale computations using standard shell elements the computing times prove to be very high.

The discussion shows, that there is a need for further research in this range. Based on above arguments we propose a shell theory and associated finite element formulation which is characterized by the following features.

(i) The underlying nonlinear shell theory is based on Reissner–Mindlin kinematic assumptions. This leads in a basic version to averaged transverse shear strains and vanishing transverse normal strains when exploiting the Green–Lagrangian strain tensor. Subsequently the displacement field is enriched by warping displacements and thickness changes using layerwise interpolation functions. Thereby an interface to arbitrary three–dimensional material laws with restriction to small strains is created.

(ii) The weak form of the boundary value problem is derived using the equilibrium equations for the stress resultants and stress couple resultants, the local equilibrium equations in terms of stresses, the geometric field equations, the constitutive equations and a constraint which enforces the correct shape of the displacement fluctuations through the thickness.

(iii) Static condensation is applied to eliminate a set of parameters on element level. The resultant quadrilateral shell element possesses the usual 5 or 6 nodal degrees of freedom. This is an essential feature since standard geometrical boundary conditions can be applied and the element is applicable also to shell intersection problems. In comparison to fully 3D computations and to FE^2 computations present formulation needs only a fractional amount of computing time.

2. FE formulation for layered shells

Layered shells of thickness h are considered in this contribution. With ξ^i we denote a convected coordinate system of the body, where for the thickness coordinate $h^- \leq \xi^3 \leq h^+$ holds. Thus, the reference surface Ω can arbitrarily be chosen related to the outer surfaces. The coordinate on the boundary $\Gamma = \Gamma_u \cup \Gamma_\sigma$ is denoted by s. The shell is loaded statically by surface loads $\bar{\mathbf{p}}$ in Ω and by boundary forces $\bar{\mathbf{t}}$ on Γ_σ. Inserting the kinematic assumptions according to Reissner and Mindlin into the Green–Lagrangian strain tensor one obtains well-known expressions for the shell strains. The membrane strains, curvatures and transverse shear strains are summarized in the vector $\boldsymbol{\varepsilon}_g$.

A displacement field $\tilde{\mathbf{u}}$ is superposed on the displacement shape of the Reissner-Mindlin theory. Its components $\tilde{u}_i$ refer to a constant element basis system, where $\tilde{u}_1, \tilde{u}_2$ denote out of plane warping displacements and $\tilde{u}_3$ thickness changes. The shape of $\tilde{\mathbf{u}}$ through the thickness is chosen as in [4]

$$\tilde{\mathbf{u}}(\xi^3) = \boldsymbol{\Phi}(\xi^3)\,\boldsymbol{\alpha}\,. \tag{1}$$

The vector $\boldsymbol{\alpha}$ is element-wise constant and contains displacements at nodes in thickness direction of the laminate. For N layers this leads to $M = 9 \cdot N + 3$ components in $\boldsymbol{\alpha}$. The interpolation matrix $\boldsymbol{\Phi}(\xi^3)$ is formulated with cubic hierarchic functions

$$\begin{aligned} &\boldsymbol{\Phi}(\xi^3) = \left[\ \phi_1\,\mathbf{1}_3 \quad \phi_2\,\mathbf{1}_3 \quad \phi_3\,\mathbf{1}_3 \quad \phi_4\,\mathbf{1}_3\ \right]\mathbf{a}^i \\ &\phi_1 = \frac{1}{2}(1-\zeta) \qquad \phi_2 = 1-\zeta^2 \qquad \phi_3 = \frac{8}{3}\zeta\,(1-\zeta^2) \qquad \phi_4 = \frac{1}{2}(1+\zeta)\,, \end{aligned} \tag{2}$$

where $-1 \leq \zeta \leq 1$ is a normalized thickness coordinate of layer i. Furthermore, $\mathbf{a}^i$ is an assembly matrix, which relates the 12 degrees of freedom of layer i to the M components of $\boldsymbol{\alpha}$.

Using admissible variations for the independent mechanical fields the variational equation as basis for the FE formulation reads

$$\int\limits_{\Omega} \delta\boldsymbol{\varepsilon}_g^T\boldsymbol{\sigma} + \delta\boldsymbol{\sigma}^T(\boldsymbol{\varepsilon}_g - \boldsymbol{\varepsilon}) + \delta\boldsymbol{\vartheta}^T\boldsymbol{\psi}\,\mathrm{d}A - \int\limits_{\Omega} \delta\mathbf{u}^T\bar{\mathbf{p}}\,\mathrm{d}A - \int\limits_{\Gamma_\sigma} \delta\mathbf{u}^T\,\bar{\mathbf{t}}\,\mathrm{d}s = 0\,. \tag{3}$$

Here, $\boldsymbol{\sigma}$ denotes the vector of independent stress resultants and $\boldsymbol{\varepsilon}$ denote physical shell strains, which enter into the material law. The vector $\boldsymbol{\psi}$ summarizes the material law, the equilibrium of higher order stress resultants and a constraint. The work conjugate vector $\boldsymbol{\vartheta} = [\boldsymbol{\varepsilon}, \boldsymbol{\alpha}, \boldsymbol{\lambda}]^T$ contains besides $\boldsymbol{\varepsilon}$ and $\boldsymbol{\alpha}$ the vector of Lagrange parameters $\boldsymbol{\lambda}$. Integration by parts and using standard arguments of variational calculus yields the associated Euler-Lagrange equations. One obtains the equilibrium of stress resultants, the geometric field equations, the local equilibrium in terms of stresses, a constraint and the static boundary conditions. The interpolation functions for the independent quantities are specified in detail in Ref. [5]. Several linear and nonlinear test examples show the effectiveness of the proposed model.

References

[1] F. Gruttmann and W. Wagner (1994). On the numerical analysis of local effects in composite structures, *Composite Struct*, **29**, 1–12.

[2] S. Klinkel, F. Gruttmann and W. Wagner (1999). A continuum based 3D–shell element for laminated structures, *Computers & Structures*, **71**, 43–62.

[3] F. Gruttmann and W. Wagner (2013). A coupled two-scale shell model with applications to layered structures, *Int. J. Num. Meth. Eng.*, **94**, 1233–1254.

[4] F. Gruttmann, W. Wagner and G. Knust (2016). A coupled global-local shell model with continuous interlaminar shear stresses, *Comp. Mech.*, **57** (2), 237–255.

[5] F. Gruttmann, G. Knust and W. Wagner (2017). Theory and numerics of layered shells with variationally embedded interlaminar stresses, *Comp. Meth. Appl. Mech. Engrg.*, **326**, 713-738.