INTERACTION BETWEEN INCLUSIONS AND CRACKS IN SMART COMPOSITES

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1. Introduction

Due to the ability of converting energy from one form (among mechanical, electric, and magnetic energies) to the other, piezoelectric and magneto-electro-elastic (MEE) materials are widely used as sensors and actuators in intelligent advanced structure design. Owing to the designable characteristics of composite laminates, a smart system constructed by composites with piezoelectric/MEE sensors and actuators is a common practical engineering design. If we treat sensors/actuators as inclusions, fracture analysis of such smart system can be studied via interaction between inclusions and cracks. With the nature of anisotropy, composite materials are usually modeled as anisotropic elastic solids. Thus, the simultaneous existence of anisotropic elastic, piezoelectric and MEE materials in the problems of inclusions and cracks is unavoidable.

In the literature, most of the inclusion problems were considered for a specific material type such as isotropic, anisotropic, piezoelectric, or MEE materials. Due to the difference of their mechanical behaviors, several different mathematical models and solution methods have been proposed. Thus, it is difficult to combine them together to deal with the possible situation that different types of materials appear simultaneously in the same structure. Among all the different methods, the Stroh formalism for two-dimensional anisotropic elasticity has been proved to preserve the same mathematical expressions for anisotropic, piezoelectric and MEE materials. By using Stroh formalism, some analytical solutions related to the inclusions, holes, or cracks in the piezoelectric or MEE materials have been shown to have the same mathematical matrix forms as their corresponding problems with anisotropic elastic materials [1].

If more than one inclusion is considered, the possibility of simultaneous existence of anisotropic, piezoelectric and MEE materials is raised. Although there are many studies on the problems with multiple inclusions, most of them still focus on a specific kind of materials. Similar situation occurs for the studies of interaction between inclusion and dislocation/crack/hole. With the special feature of preserving the same solution form by Stroh formalism, in this paper we develop an adaptable adjustment technique and apply it to the dislocation superimposed method (DSM) [2] and the boundary-based finite element method (BFEM) [3] to deal with fracture analysis of composites in which the piezoelectric and MEE inclusions may exist simultaneously. The proposed technique intends to adjust the matrix dimension of Stroh formalism to include the piezoelectric and/or magnetic effects from the ordinary elasticity matrices according to the real situation of the inclusion problem. To show the correctness of the proposed method, several examples are implemented by using the extended DSM or BFEM, and compared with the solutions calculated by the commercial finite element software ANSYS. Since ANSYS does not provide the function for the general MEE materials, for the purpose of comparison with ANSYS only special cases of MEE such as piezoelectric or piezomagnetic are calculated. The general cases of MEE are verified by the comparison between DSM and BFEM provided in this paper, or through its mechanical behavior.

2. Numerical examples

In order to study the interaction between an elliptical inclusion and a crack, we consider a fiber-reinforced composite with an elliptical MEE inclusion and an inclined crack subjected to a uniform tension $\sigma_{22}^{\infty}=1\text{MPa}$ at infinity (Figure 1) . The properties of composites are

$$E_1=181\text{GPa},\ E_2=10.3\text{GPa},\ G_{12}=7.17\text{GPa},\ \nu_{12}=0.28,$$

where the symbols E, G and ν denote, respectively, Young's modulus, shear modulus and Poisson's ratio, and the subscripts 1 and 2 denote, respectively, the fiber direction and the direction transverse to the fiber.

The MEE inclusion is made by mixing the fiber-reinforced composite with piezoelectric $BaTiO_3$ and piezomagnetic $CoFe_2O_4$, whose properties are

$$\begin{aligned}
&C_{11} = C_{33} = 166\text{GPa},\ C_{12} = C_{23} = 78\text{GPa},\ C_{13} = 77\text{GPa},\\
&C_{22} = 161\text{GPa},\ C_{44} = C_{66} = 43\text{GPa},\ C_{55} = 44.5\text{GPa},\\
&e_{21} = e_{23} = -4.4\text{C/m}^2,\ e_{22} = 18.6\text{C/m}^2,\ e_{16} = e_{34} = 11.6\text{C/m}^2,\\
&\omega_{11} = \omega_{33} = 11.2\times10^{-9}\text{C/V},\ \omega_{22} = 12.6\times10^{-9}\text{C/V},\\
&q_{21} = q_{23} = 580.3\text{N/Am},\ q_{22} = 699.7\text{N/Am},\ q_{16} = q_{34} = 550\text{N/Am},\\
&m_{11} = m_{33} = 5\times10^{-12}\text{Ns/VC},\ m_{22} = 3\times10^{-12}\text{Ns/VC},\\
&\xi_{11} = \xi_{33} = 5\times10^{-6}\text{Ns}^2/\text{C}^2,\ \xi_{22} = 10^{-5}\text{Ns}^2/\text{C}^2,
\end{aligned}$$

in which C_{ij}, e_{ij}, ω_{jk}, q_{ij}, m_{ij}, and ξ_{ij} are, respectively, the elastic constants, piezoelectric stress constants, dielectric permittivity constants, piezomagnetic constant, magneto-electric constant and permeability constant. Table 1 shows the normalized stress intensity factors, $K_I / \sigma_0\sqrt{\pi l}$ and $K_{II} / \sigma_0\sqrt{\pi l}$ where $\sigma_0 = \sigma_{22}^{\infty}$, of the crack tip *A* (Figure 1) with various *d* and α_c. From Table 1 we see that the results obtained by DSM and BFEM are quite close to each other. When the distance between inclusion and crack is larger, the values of stress intensity factors go to the ones for a crack in a homogeneous material, which are $K_I = \sigma_0 \cos^2\alpha_c\sqrt{\pi l}$ and $K_{II} = \sigma_0 \sin\alpha_c \cos\alpha_c\sqrt{\pi l}$. Moreover, the stress intensity factors decrease when the crack is closer to the inclusion (see for example the variation of K_I with *d* when $\alpha_c = 0^o$). It is now not easy to judge the consistency with the conclusion made for isotropic materials [4] – the presence of harder/softer inclusions will reduce/enhance the stress intensity of the crack. Thus, further examples and studies are necessary for smart composites.

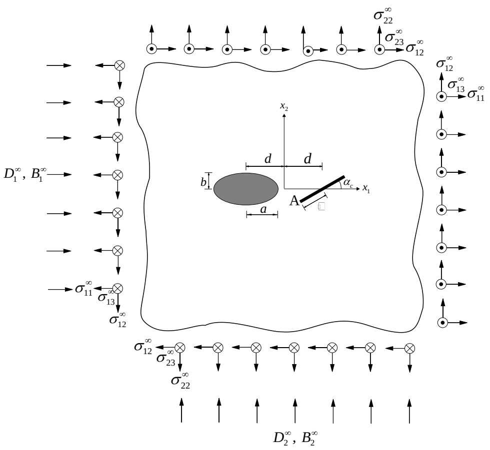

Figure 1. An infinite plate with an elliptical inclusion and a crack under uniform load at infinity. (*a*=2 m, *b*=1 m, l =1 m)

Inclusion	*d/a*	α_c(°)	$K_I / \sigma\sqrt{\pi l}$		$K_{II} / \sigma\sqrt{\pi l}$	
			DSM	BFEM	DSM	BFEM
MEE	6	0	0.9985	0.9974	0	0
		45	0.4969	0.4970	0.5014	0.5003
		60	0.2463	0.2469	0.4341	0.4331
	4	0	0.9962	0.9950	0	0
		45	0.4933	0.4933	0.5025	0.5012
		60	0.2425	0.2430	0.4348	0.4336
	2	0	0.9814	0.9797	0	0
		45	0.4801	0.4766	0.5011	0.5017
		60	0.2322	0.2313	0.4332	0.4310

Table1. Stress intensity factor of crack tip A

Acknowledgments The authors would like to thank Ministry of Science and Technology, TAIWAN, R.O.C for support through Grants MOST 104-2221-E-006-138-MY3.

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