MODEL EQUATIONS OF DIFFUSIVE MICROSTRETCH THERMOELASTICITY WITH MICROTEMPERATURES AND MICROCONCENTRATIONS

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1. Motivation

This paper is dealing with the linear theory of microstretch thermoelasticity, a theory that is part of the mechanics of generalized continua and which accurately describes the behaviour of materials with microstructure. The concept of generalized continua was introduced by the Cosserat brothers in 1909, who considered a micropolar continuum to be a collection of interconnected particles in the form of small rigid bodies that have three additional rotational degrees of freedom besides the three translational degrees of freedom from classical continuum mechanics. Cosserat thermoelasticity was generalized to microstretch thermoelasticity by Eringen in 1990 [6] by including the effect of axial stretch during the rotation of molecules.

In this paper, we prove the well-posedness of the mathematical model introduced in [3] for the anisotropic case. The idea is to verify the assumptions from the Lumer-Phillips corollary to the Hille-Yosida theorem. As in [2], the theory we derived is closer to the realistic constitutive structure of solids since it describes thermal-diffusion interactions at the macroscopic and microscopic levels. Given the increasing interest in nanomaterials, it is important to take into consideration both the microtemperatures and the microconcentrations of the nanoparticles. The concept of microconcentrations is a novel one, introduced for the first time in the theory of thermoelasticity in [2] and for the first time in mechanics of generalized continua in [3].

2. Basic equations

The equations of diffusive microstretch thermoelasticity with microtemperatures and microconcentrations [3]

$$
\begin{array}{lll}
t_{ji,j} + \rho f_i = \rho \ddot{u}_i & \rho T_0 \dot{S} = q_{i,i} + \rho s & \\
h_{k,k} + g + \rho l = J\ddot{\varphi} & \rho\dot{\varepsilon}_i = q_{ji,j} + q_i - Q_i + \rho G_i & \dot{C} = \eta_{i,i} \\
m_{ji,j} + \varepsilon_{irs} t_{rs} + \rho g_i = I_{ij}\ddot{\varphi}_j & \rho\dot{\omega}_i = \eta_{ji,j} + \eta_i - \tilde{\sigma}_i &
\end{array}
\tag{1}
$$

In the equations above, u_i is the displacement vector field, φ is the microdilatation function, φ_i is the microrotation vector, t_{ij} is the stress tensor, ρ is the reference mass density, f_i is the body force, h_j is the microstretch vector, g is the internal body force, l is the external microstretch body load, m_{ij} is the couple stress tensor, g_i is the body couple density, C is the concentration, η_i is the flux vector of mass diffusion, S is the microentropy. According to [3], the constitutive equations of the mathematical model for the anisotropic case are

$$
\begin{aligned}
& t_{ij} = A_{ijrs}e_{rs} + B_{ijrs}\kappa_{rs} + D_{ij}\varphi + F_{ijk}\zeta_k + L_{ijk}T_k - a_{ij}\theta + d_{ij}C \\
& m_{ij} = B_{rsij}e_{rs} + C_{ijrs}\kappa_{rs} + E_{ij}\varphi + G_{ijk}\zeta_k + M_{ijk}T_k - b_{ij}\theta + f_{ij}C \\
& h_i = F_{rsi}e_{rs} + G_{rsi}\kappa_{rs} + A_{ij}\zeta_j + B_i\varphi - N_{ij}T_j - d_i\theta + \tilde{f}_iC \\
& g = -D_{ij}e_{ij} - E_{ij}\kappa_{ij} - B_i\zeta_i - \xi\varphi - R_iT_i + F\theta - \tilde{g}_1C \\
& \rho S = a_{ij}e_{ij} + b_{ij}\kappa_{ij} + d_i\zeta_i + F\varphi + b_iT_i + a\theta + \varpi C \\
& \rho\varepsilon_i = L_{rsi}e_{rs} + M_{rsi}\kappa_{rs} - N_{ji}\zeta_j + R_i\varphi - B_{ij}T_j - b_i\theta - R_{ij}C_j \\
& P = d_{ij}e_{ij} + f_{ij}\kappa_{ij} + \tilde{f}_i\zeta_i + \tilde{g}_1\varphi - \varpi\theta + \varrho C \qquad \rho\omega_i = -C_{ij}C_j - R_{ji}T_j
\end{aligned}
\tag{2}
$$

Here, T_0 is the absolute temperature in the reference configuration, q_i is the heat flux vector, s is the heat supply per unit mass, ε_i is the first moment of energy vector, q_{ij} is the first heat flux moment tensor, Q_i is the microheat flux average, G_i is the first heat supply moment vector, η_{ij} is the first mass diffusion flux moment tensor, $\tilde{\sigma}_i$ is the micromass diffusion flux average, P is the particle chemical potential, T is the absolute temperature, $\theta = T - T_0$, T_i are the microtemperatures, C_i are the microconcentrations and ε_{ijk} is the alternating symbol.

3. Well-posedness

We assume null boundary conditions. We introduce the notations $\dot{u}_i = v_i$, $\dot{\varphi}_i = \psi_i$ and $\dot{\varphi} = \psi$. Let $\mathcal{H} = \left\{(u_i, v_i, \varphi, \psi, \varphi_i, \psi_i, \theta, T_i, P, C_i) : u_i, \varphi_i \in \mathbb{W}_0^{1,2}(\Omega), v_i, \psi_i, T_i, C_i \in \mathbb{L}^2(\Omega), \psi, \theta, P \in L^2(\Omega), \varphi \in W_0^{1,2}(\Omega)\right\}$ where $W_0^{1,2}(\Omega)$, $L^2(\Omega)$ are the familiar Sobolev spaces and $\mathbb{W}_0^{1,2}(\Omega) = \left[W_0^{1,2}(\Omega)\right]^3, \mathbb{L}^2(\Omega) = \left[L^2(\Omega)\right]^3$. The boundary initial value problem can be transformed into the following equation in the Hilbert space $\mathcal{H}$

$$\frac{\mathrm{d}\mathcal{U}}{\mathrm{d}t} = \mathcal{A}\mathcal{U}(t) + \mathcal{F}(t) \qquad \mathcal{U}(0) = \mathcal{U}_0 \tag{3}$$

where $\mathcal{U} = (u_i, v_i, \varphi, \psi, \varphi_i, \psi_i, \theta, P, T_i, C_i)$, $\mathcal{U}_0 = \left(u_i^0, v_i^0, \varphi^0, \psi^0, \varphi_i^0, \psi_i^0, \theta^0, P^0, T_i^0, C_i^0\right)$ is the vector of initial conditions and $\mathcal{A}$ is a certain matrix operator on $\mathcal{H}$.

Lemma 3.1 *In the case of diffusive microstretch thermoelasticity with microtemperatures and microconcentrations and for every $\mathcal{U} \in \mathcal{D}(\mathcal{A})$, the operator $\mathcal{A}$ satisfies the inequality*

$$\langle \mathcal{A}\mathcal{U}, \mathcal{U}\rangle \leq 0 \tag{4}$$

in a suitably introduced inner product in $\mathcal{H}$.

Lemma 3.2 *In the case of diffusive microstretch thermoelasticity with microtemperatures and microconcentrations, and for Id the identity operator in $\mathcal{H}$, the operator $\mathcal{A}$ has the property that*

$$Range(Id - \mathcal{A}) = \mathcal{H} \tag{5}$$

Theorem 3.1 *The operator $\mathcal{A}$ generates a semigroup of contractions in $\mathcal{H}$.*

This result proves that in the motion following any sufficiently small change in the external system, the solution of the initial boundary value problem is everywhere arbitrary small in magnitude. Now that we proved that the mathematical model is well-posed by means of the semigroup of linear operator theory, the asymptotic behaviour of solutions and the effect of a concentrated heat source can be studied.

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